Dynamics of the Lightning Discharge Using Generalized Travelling Current Source Return Stroke Model

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Abstract:

- Generalized lightning traveling current source return stroke model (GTCSM) has been used to examine the dynamics of lightning channel corona sheath surrounding thin channel core.
- The corona sheath radius and velocity of the corona sheath radial expansion are calculated based on assumed line charge distributions.
- Application to the Traveling Current Source, the Diendorfer-Uman (DU) and modified DU model.

Channel-base current

$$i_0(t) = (I/\eta)\,t^n/(t^n + \tau_1^n)\exp(-t/\tau_2)$$

Table 1. Typical lightning current quantities of the negative return stroke measured at the top of the tower (according to Berger et al, 1975, Anderson & Erikson, 1980)

i_{0max} (kA)	$(di/dt)_{max}$(kA/μs)	τ_p (μs)	Q_0 (mC)
13	74	0.235	50

Table 2. The channel-base current parameters according to Eq.(3) and the measurements given in Table 1.

I (kA)	η	n	τ_1 (μs)	τ_2(μs)	τ_d(μs) (DU)	v (m/μs)
13	0.84	5	0.26	3.3	0.6	100

GTCSM

- The current at the channel base and the initial charge distribution along the channel are considered as known. The channel discharge function *f* is calculated.

$$q'(z,t) = q'_0(z,t)\, f(z, t - z/v), \quad t \geq z/v$$

$$f(z, t - z/v) = 1 + \int_0^{t-z/v} f_1(z,\xi)\, d\xi$$

$$f_1 = F^{-1}\left[I_0(s) / Q'_0(s/v^*) \right], \quad v^* = vc/(v+c)$$

$$i(z,t) = \int_z^{h_{az}} q'_0(\xi) \frac{\partial}{\partial t} f(z, t - \xi/v^* + z/c)\, d\xi \,,$$

$$h_{az} = v^*(t + z/c)$$

$$(1)\quad f(z,u=0)=1,\qquad u=t-z/v$$

$$(2)\quad f(z,u\to\infty)=0$$

$$(3)\quad f(z,u\ge 0)\ge 0$$

$$(4)\quad \partial f(z,u)/\partial u|_{u\ge 0}<0$$

- Assumed analytical form of the leader charge distribution function

$$q_0^{'}(z)=Q_{01}^{'}\left[g(z)+\lambda_d(z)\frac{dg}{dz}\right],\quad g(z)=\frac{z^m}{z^m+\lambda_1^m}\exp(-z/\lambda_2) \qquad (1)$$

Table 3. The values of the parameters of the leader channel charge distribution according to Eq.(1)

Curve	Q'_{01} (mC/m)	λ_1 (m)	λ_2 (m)	λ_d (m)	m
(a)	0.231	19.6	247	45	1
(b)	0.195	5	247	45	3
(c)	0.541	10	98	0	0
(DU)	0.206	19.5	247.5	45	5

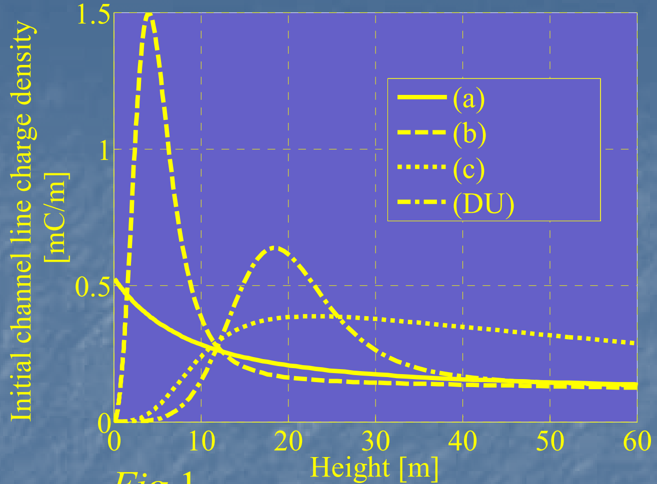

Fig.1.

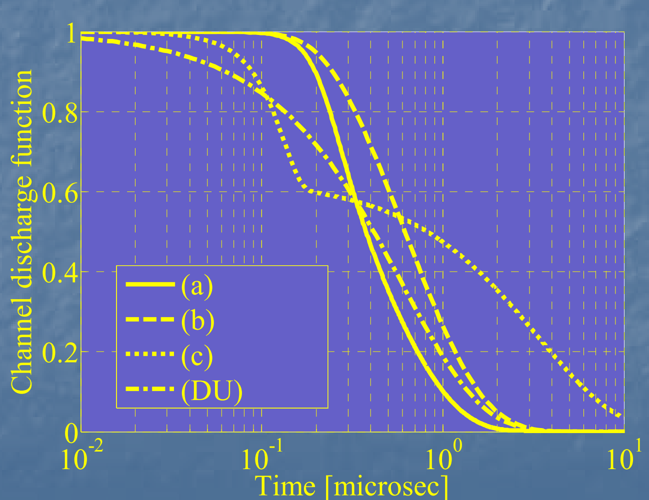

Fig.2.

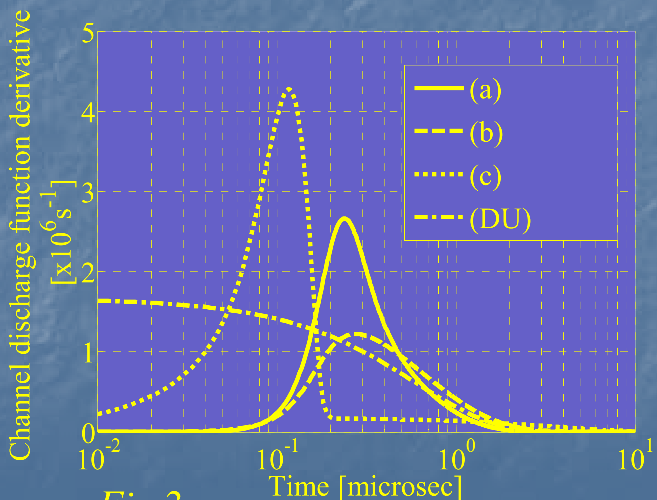

Fig.3.

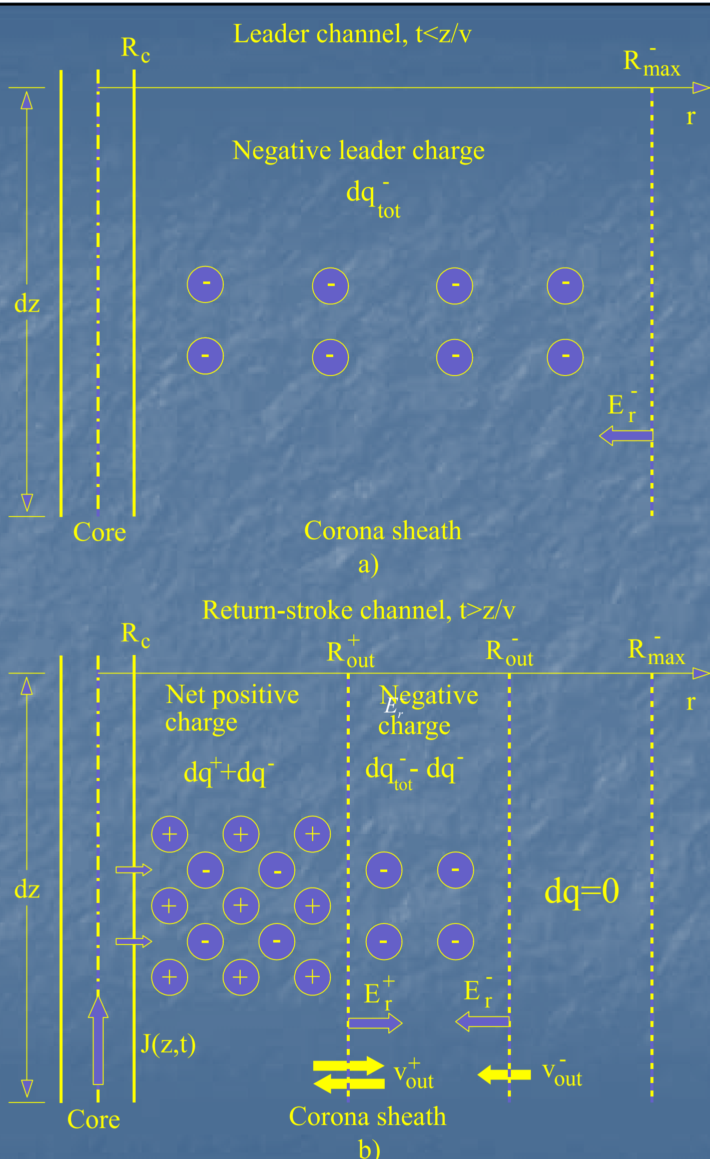

Fig. 4. a) The structure of the corona sheath just prior to the return stroke stage, $R^-_{\max}$ is the maximum radial extent of the negative leader corona sheath, E^-_r is negative breakdown electric field at the outer surface of the corona sheath, dq^-_{tot} is the total negative leader charge (Maslowski and Rakov, 2006).

Fig. 4. b) The structure of the corona sheath during the return stroke stage, dq^- and dq^+ are negative and positive charges within R^+_{out}, respectively, where R^+_{out} is the maximum radial extent of the positive return stroke charge, R^-_{out} is the radial extent of the negative leader corona sheath during the return stroke, E^+_r and E^-_r are positive and negative breakdown electric fields at the radial distances R^+_{out} and R^-_{out}, respectively.

$E^+_r = \left| E^-_r \right| = 1\,\mathrm{MV/m}$

- In accordance with the above assumed mechanism of discharge and taking into account the GTCSM the line charge density in the channel can be expressed as

$$q'(z,u) = q_0'(z) \cdot f(z,u) = q_0'(z) + dq^+(z)/dz\,,$$

where

$$dq^+(z) = q_0^+(z) f^+(z,u)\,dz\,,\; u = t - z/v\,,$$

$$f^+ = 1 - f\,,\;\; q_0^+(z) = -q_0'(z).$$

f^+ should satisfy the following features

$$a)\, f^+(u=0) = 0, \quad b)\, f^+(u>0) > 0,$$

$$c)\,(df^+/du)_{u\geq 0} \geq 0 \;\; d)\, f^+(u \to \infty) = 1.$$

Outer corona sheath containing net negative charge

Applying Gauss' law on the elementary part of the channel, Fig.4b, one obtains

$$R_{out}^{-} = 2B(1 - f^{+})\ , \quad B = q_0^{+} / (4\pi\varepsilon_0 \left|E_r^{-}\right|)\ ,$$

$$v_{out}^{-} = dR_{out}^{-} / du = -2B\,(df^{+} / du)\ .$$

$$q_0^{+} = q_{0\max}^{+}\ , Fig.1$$

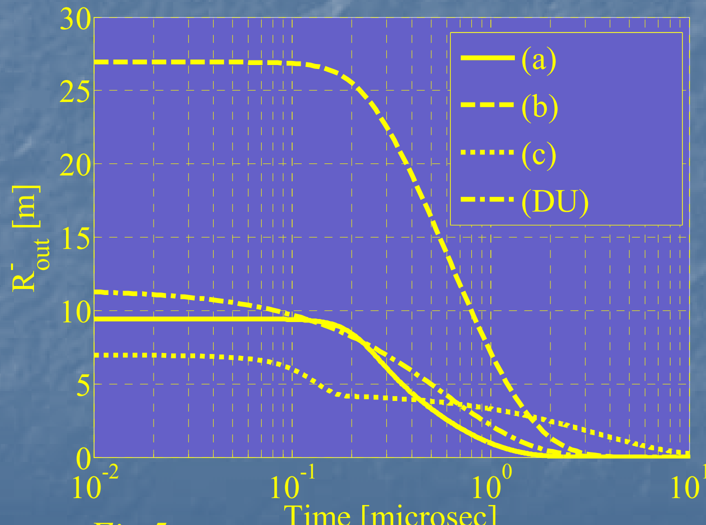

Fig.5.

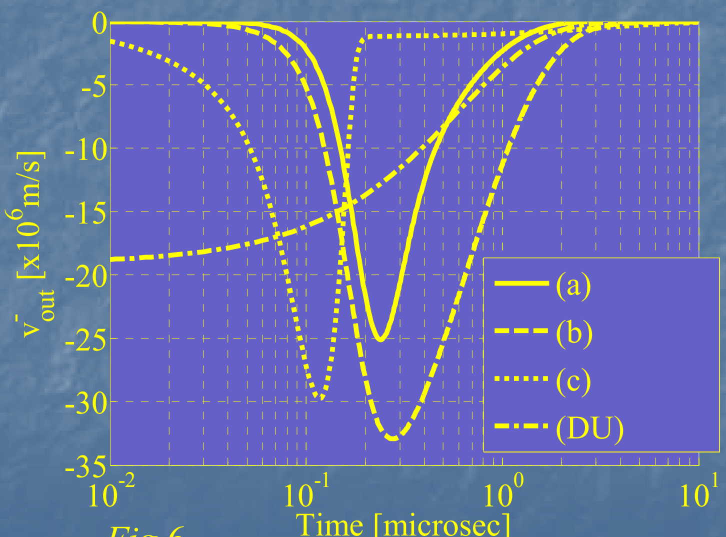

Fig.6.

Inner corona sheath containing net positive charge

Inner corona sheath radius can be obtained solving the equation

$$E_r^+ = \frac{q_0^+}{2\pi\varepsilon_0}\left[\frac{f^+}{R_{out}^+} - \frac{R_{out}^+}{(R_{out}^-)^2}\right], \rightarrow R_{out}^+ = 2B(1-f^+)f^+,$$

$$v_{out}^+ = 2B(df^+/du)(1-2f^+).$$

$$q_0^+ = q_{0\max}^+, Fig.1$$

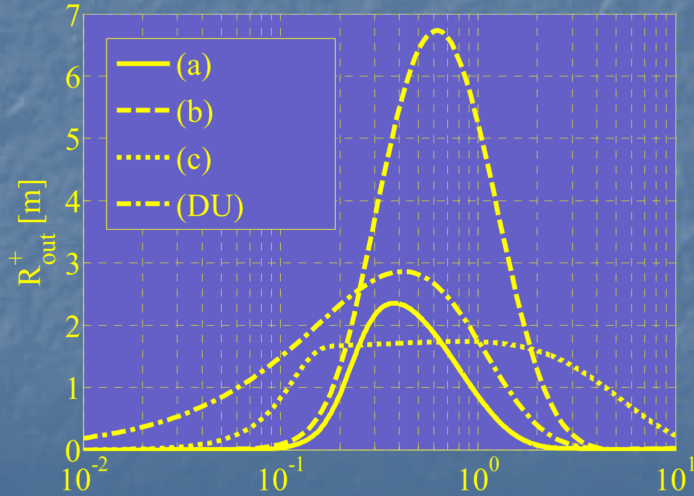

Fig.7.

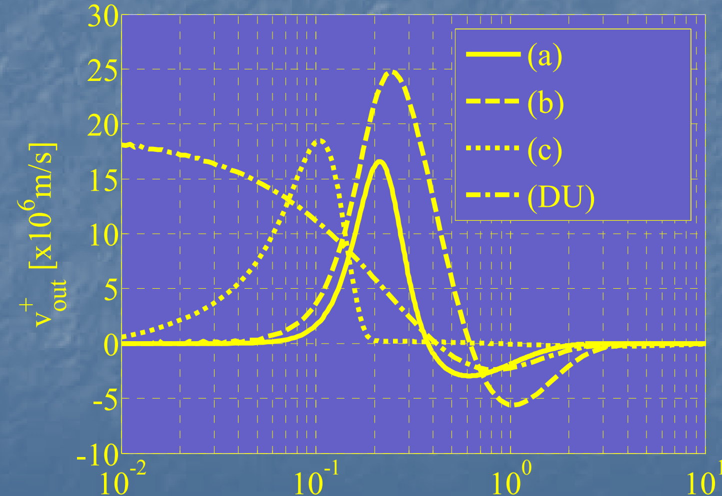

Fig.8.

Application to the Traveling Current Source (Heidler 1985), Dindorfer-Uman (Dindorfer and Uman, 1990) and modified Dindorfer-Uman (Thottappillil-Uman, 1994) model

Table 4. Inner and outer radii and their velocities according to the GTCSM during RS

RS Model	Channel discharge function $f(z,u)$, $u = t - z/v$	Deposited charge density $q_0^+ = B4\pi\varepsilon_0 \lvert E_r^- \rvert$	R_{out}^+	v_{out}^+	R_{out}^-	v_{out}^-
TCS	$1 - h(u)$	$i_0(z/v^*)/v^*$	$2B(1-h)h$	$2B\delta(1-2h)$	$2B(1-h)$	$-2B\delta$
DU	$\exp(-\frac{u}{\tau})$, $\tau = const.$	$\frac{i_0(z/v^*)}{v^*} + \left.\frac{di_0}{dt}\right\rvert_{t=\frac{z}{v^*}} \frac{\tau_d}{v^*}$	$[1-\exp(-\frac{u}{\tau})]\times$ $\times 2B\exp(-\frac{u}{\tau})$	$[-1+2\exp(-\frac{u}{\tau})]\times$ $\times\frac{2B}{\tau}\exp(-\frac{u}{\tau})$	$2B\times$ $\times\exp(-\frac{u}{\tau})$	$-\frac{2B}{\tau}\times$ $\times\exp(-\frac{u}{\tau})$
MDU	$\exp(-\frac{u}{\tau(z)})$	$\frac{i_0(z/v^*)}{v^*} + \left.\frac{di_0}{dt}\right\rvert_{t=\frac{z}{v^*}} \frac{\tau_d(z)}{v^*}$	$[1-\exp(-\frac{u}{\tau(z)})]\times$ $\times 2B\exp(-\frac{u}{\tau(z)})$	$[-1+2\exp(-\frac{u}{\tau(z)})]\times$ $\times\frac{2B}{\tau(z)}\exp(-\frac{u}{\tau(z)})$	$2B\times$ $\times\exp(-\frac{u}{\tau(z)})$	$-\frac{2B}{\tau(z)}\times$ $\times\exp(-\frac{u}{\tau(z)})$

i_0 – Channel-base current, $v^* = vc/(v+c)$, h – Unit function, δ-Dirac function