Review of Equivalent Methods for Computing Electromagnetic Fields From an Extending Lightning Discharge

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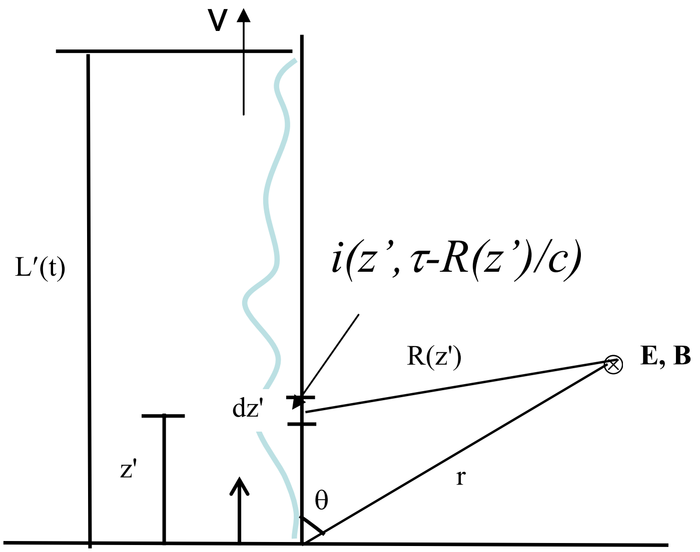
Outline

- Description of the problem
- Dipole and monopole methods for field calculations
- Non-uniqueness of field components
- Fields in terms of apparent charge (third method)

Description of the problem

- Lightning return stroke speed 1-2x10^8 m/s
- Current rise time < 10^{-6} s
- Distributed source fast changing in both space and time
- Methods of finding exact expressions for remote electric and magnetic fields

$L'(t) < v.t$



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Proper treatment of retardation effects

- Limits of integration

$$t = \frac{L'(t)}{v} + \frac{R(L')}{c} \qquad R(L') = \sqrt{r^2 + L'^2(t) - 2L'(t) r \cos\theta}$$

$$L'(t) = \frac{r}{1-(v^2/c^2)}\left(-\frac{v^2}{c^2}\cos\theta + \frac{vt}{r} - \frac{v}{c}\sqrt{\left(1-\frac{v^2}{c^2}\right) + \frac{v^2t^2}{r^2} + \frac{v^2}{c^2}\cos^2\theta - \frac{2vt}{r}\cos\theta} \right)$$

$$\frac{dL'}{dt} = \frac{v}{1-\frac{v}{c}\cos\theta(L')}$$, apparent speed of return stroke wavefront

Very far away $L'(t) = \dfrac{v}{1-\frac{v}{c}\cos\theta}\cdot(t - r/c)$

F-factor, $F = \left[1-\dfrac{v}{c}\cos\theta\right]^{-1}$

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Dipole and monopole methods for field calculations - 1

- Dipole method

(The Lorentz condition approach)

$$\overline{A}(r,\theta,\tau) = \frac{1}{4\pi\varepsilon_0 c^2}\int_0^{L'(\tau)} \frac{i(z',\tau - R(z')/c)}{R(z')}\hat{z}dz'$$

$$\phi(r,\theta,t) = -c^2 \int_{r/c}^{t} \nabla \bullet \overline{A}\, d\tau$$

(Lorentz condition)

$$\overline{E} = -\nabla\phi - \frac{\partial \overline{A}}{\partial t}$$

$$\overline{B} = \nabla \times \overline{A}$$

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Different analytical methods for field calculations - 2

- The monopole method

(The continuity equation approach)

$$\overline{A}(r,\theta,\tau) = \frac{1}{4\pi\varepsilon_0 c^2}\int_0^{L'(\tau)} \frac{i(z',\tau - R(z')/c)}{R(z')}\hat{z}\,dz'$$

$$\frac{\partial \rho^*(z',t-R(z')/c)}{\partial t} = -\left.\frac{\partial i(z',t-R(z')/c)}{\partial z'}\right|_{t-R(z')/c=const.}$$

$$\phi(r,t) = \frac{1}{4\pi\varepsilon_0}\frac{Q(t-r/c)}{r} + \frac{1}{4\pi\varepsilon_0}\int_0^{L'(t)}\frac{1}{R(z')}\rho^*(z',t-R(z')/c)\,dz'$$

$$\overline{E} = -\nabla\phi - \frac{\partial \overline{A}}{\partial t}$$

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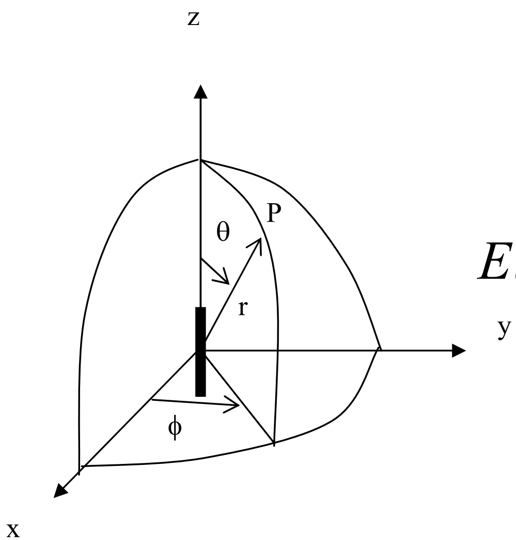
Electric fields from a dipole



Static

$$E_r = \frac{Z_0}{2\pi} m_E \cos\theta \left(\frac{1}{r^2} + \frac{1}{j\beta r^3} \right) e^{-j(\beta r - \omega t)}$$

$$E_\theta = \frac{-jZ_0\beta}{4\pi} m_E \sin\theta \left(\frac{1}{r} + \frac{1}{j\beta r^2} - \frac{1}{\beta^2 r^3} \right) e^{-j(\beta r - \omega t)}$$

$$H_\phi = \frac{j\beta}{4\pi} m_E \sin\theta \left(\frac{1}{r} + \frac{1}{j\beta r^2} \right) e^{-j(\beta r - \omega t)}$$

Radiation

Induction or intermediate

Are the field components unique?

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Electric fields from line sources

(Line source – several dipoles connected end to end)

Is it possible to define static, induction and radiation components uniquely?

How do we define static field? 1/distance^3 ?

How do we define radiation field? 1/distance ?

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Sample calculation using the two methods (lightning return stroke field at ground)

$$i(z',t) = i(0, t - z'/v)$$

Dipole method

$$E_V(r,t) = \frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{2-3\sin^2\alpha(z')}{R^3(z')}\int_{t_b}^{t} i\,(z', \tau - R(z')/c)d\tau\, dz'$$
$$+ \frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{2-3\sin^2\alpha(z')}{cR^2(z')} i(z', t - R(z')/c)\, dz'$$
$$- \frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{\sin^2\alpha(z')}{c^2 R(z')}\frac{\partial i(z', t - R(z')/c)}{\partial t} dz'$$

Monopole method

$$E_V(r,t) = -\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{z'}{R^3(z')}\rho^*(z', t - R(z')/c)\, dz'$$
$$- \frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{z'}{cR^2(z')}\frac{\partial \rho^*(z', t - R(z')/c)}{\partial t} dz'$$
$$- \frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{1}{c^2 R(z')}\frac{\partial i(z', t - R(z')/c)}{\partial t} dz'$$

Static

Induction

Radiation

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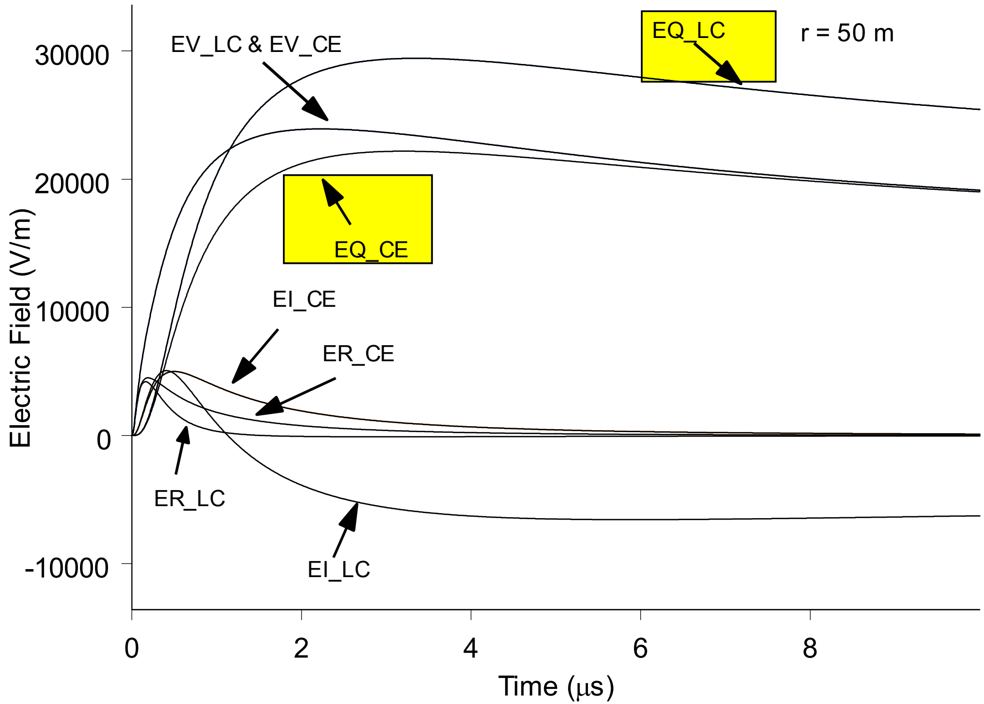
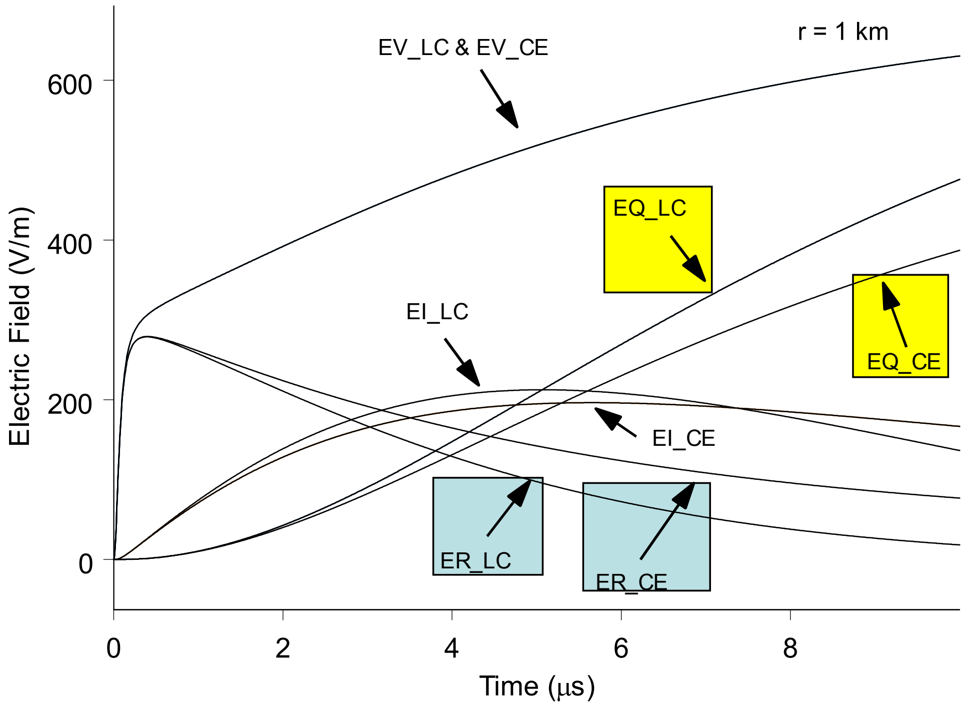
Non-uniqueness of field components (Numerical example)

R = 50 m



Non-uniqueness of field components (Numerical example)

R = 1000 m



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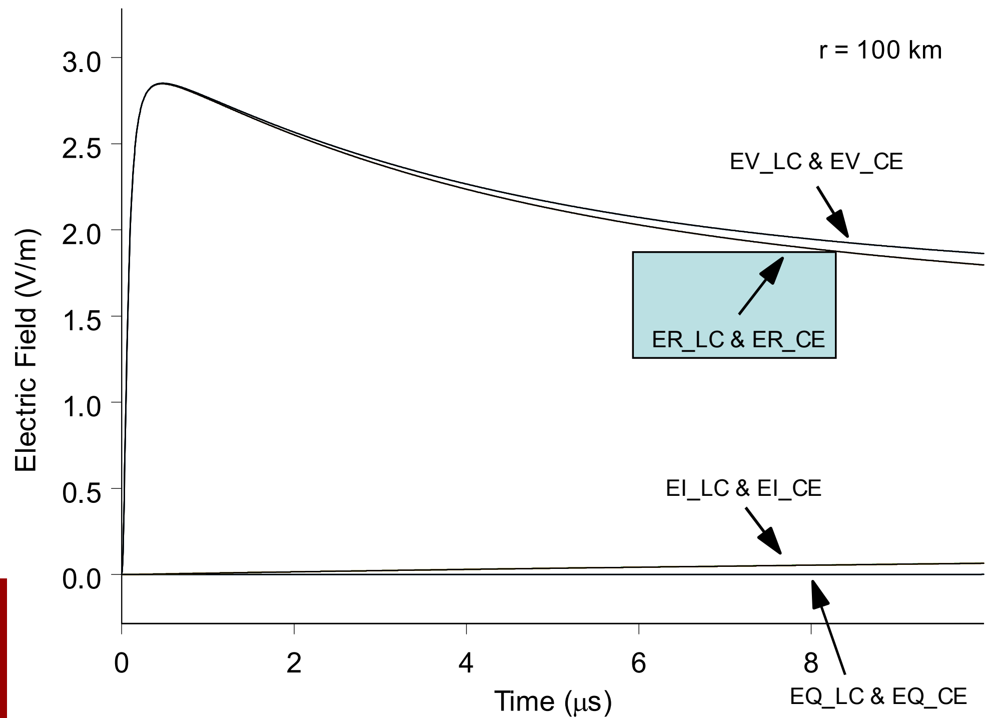
Non-uniqueness of field components (Numerical example)

R = 100 000 m



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Electric field at ground plane

(Dipole method)

- Both the gradient of the **scalar potential** and the time derivative of the **vector potential** contribute to the **radiation field** term.

- Time derivative of the **vector potential** contribute to the induction field term.

(Monopole method)

- **Radiation term** is completely given by the time derivative of the **vector potential**.

- Electrostatic and induction terms are given completely by the gradient of the **scalar potential**

No one-to-one correspondence between field components

However, total field is the same

Relation between retarded current and retarded charge

?

$$\frac{\partial \rho^*(z',t-R(z')/c)}{\partial t} = -\left.\frac{\partial i(z',t-R(z')/c)}{\partial z'}\right|_{t-R(z')/c=const.}$$

OR

$$\frac{\partial \rho(z',t-R(z')/c)}{\partial t} = -\frac{\partial i(z',t-R(z')/c)}{\partial z'}$$

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Relation between two definitions of retarded charge density

$$\rho(z',t-R(z')/c) \quad = \quad \rho^*(z',t-R(z')/c)+\frac{z'-r\cos\theta}{cR(z')}i(z',t-R(z')/c)$$

$$\left(\frac{z'-r\cos\theta}{cR(z')}=-\frac{\partial(R/c)}{\partial z'}\right)$$

Local charge density at retarded time

Local charge density at retarded time as 'seen' by remote observer

(apparent charge density)

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Relation between ‘apparent charge density’ and retarded current

$$\rho(z',t-R(z')/c) = -\frac{d}{dz'}\int_{z'/v+R(z')/c}^{t} i(z',\tau-R(z')/c)\,d\tau$$

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Fields at ground in terms of apparent charge density

$$E_z(r,t) = -\frac{1}{2\pi\varepsilon_0}\int_0^{L'(t)} \frac{z'}{R^3(z')}\rho(z',t-R(z')/c)dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\int_0^{L'(t)} \left(\frac{3}{2}\frac{z'}{cR^2(z')} - \frac{1}{2}\frac{\tan^{-1}(z'/r)}{cr}\right)\frac{\partial\rho(z',t-R(z')/c)}{\partial t}dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\left(\frac{3}{2}\frac{L'(t)}{cR^2(L')} - \frac{1}{2}\frac{\tan^{-1}(L'(t)/r)}{cr}\right)\rho(L',\frac{L'(t)}{v})\frac{dL'(t)}{dt}$$

$$-\frac{1}{2\pi\varepsilon_0}\int_0^{L'(t)} \frac{z'}{c^2R(z')}\frac{\partial^2\rho(z',t-R(z')/c)}{\partial t^2}dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\frac{L'(t)}{c^2R(L')}\frac{\partial}{\partial t}\left[\rho(L'(t),\frac{L'(t)}{v})\frac{dL'(t)}{dt}\right]$$

$$-\frac{1}{2\pi\varepsilon_0}\frac{r^2}{c^2R^3(L')}\rho(L',\frac{L'(t)}{v})\left(\frac{dL'}{dt}\right)^2$$

⋮

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Inferences

- Individual field components - static, induction, and radiation - are not unique
- Total electric field is unique
- Differences between field components are significant at close distances and negligible at far distances
- Caution has to be exercised in interpreting measurement results or in making approximations in calculations

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