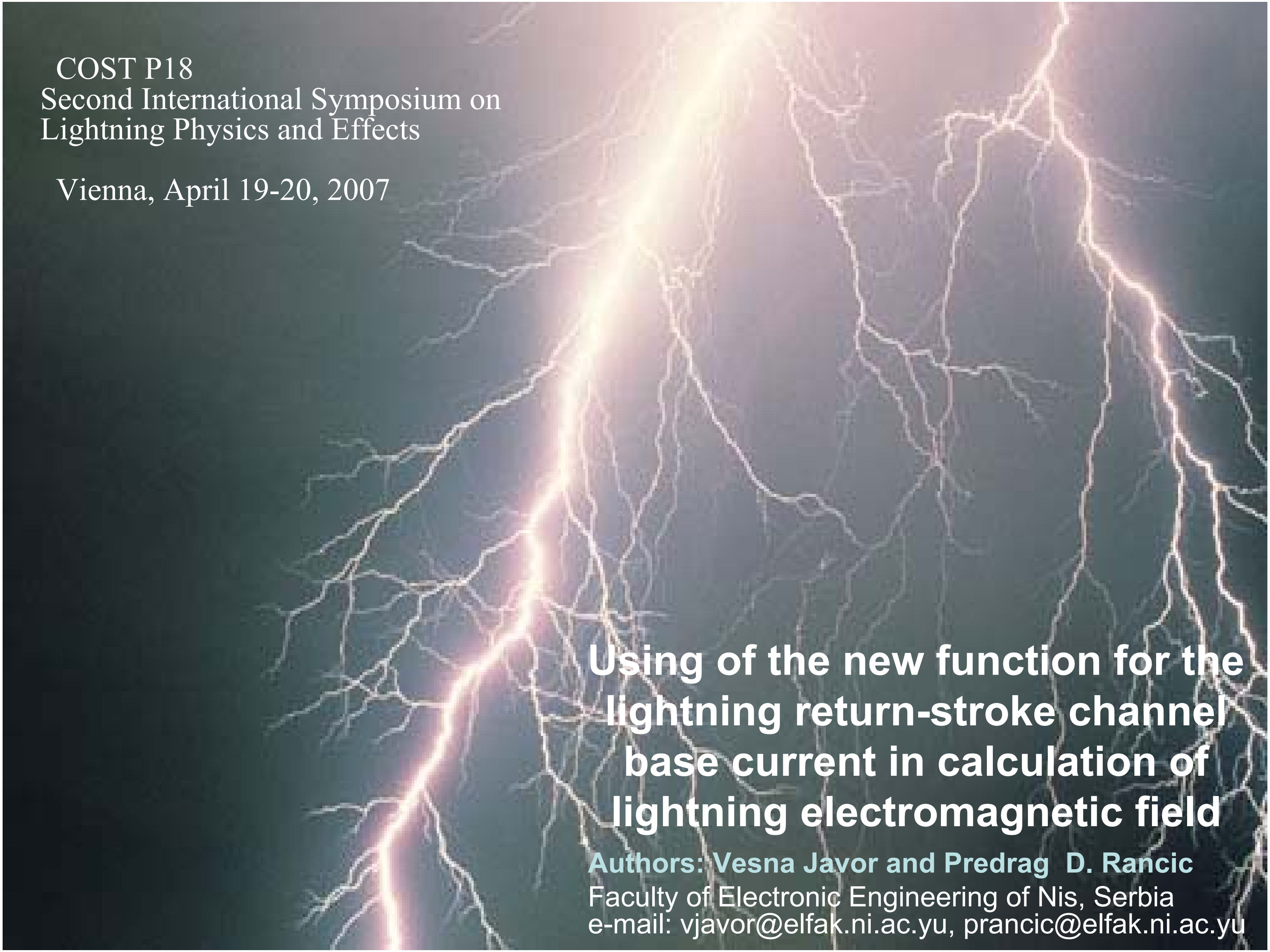

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Using of the new function for the lightning return-stroke channel base current in calculation of lightning electromagnetic field

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- **New return stroke channel-base current (CBC)**
- **Parameters of the CBC function**
- **Some comments on the CBC parameters choice**
- **The influence of the total lightning channel height on LEMF**

CONCLUSION

INTRODUCTION

For engineering models of lightning return-stroke current, implying a lumped current source at the base, there are different functions proposed for the channel-base current.

One new function for the return-stroke channel-base current and its characteristics are described in this paper.

The function can be used for approximating different desired characteristics of the lightning return-stroke channel-base current.

Lightning channel modeling

It is supposed that the **lightning channel** is vertical, straight, without branching and without reflections from the end of channel at height 2600-7000m and with pulse propagation velocity $c/3 \div c/2$

Types of lightning channel current models:

1. Physical (gas-dynamical)
2. Electromagnetic
3. Distributed circuit models
4. Engineering models

Engineering models:

- TCS model (travelling current source model)
- TL model (transmission line model)
- MTL model (modified TL model) which can be:
  - MTLL (with linear decay with height)
  - MTLE (with exponential decay with height)
  - MTLD model (modified model, with current distortion along the channel included in TCS or MTLL model),

etc.

Vertical mast antenna above perfectly conducting ground

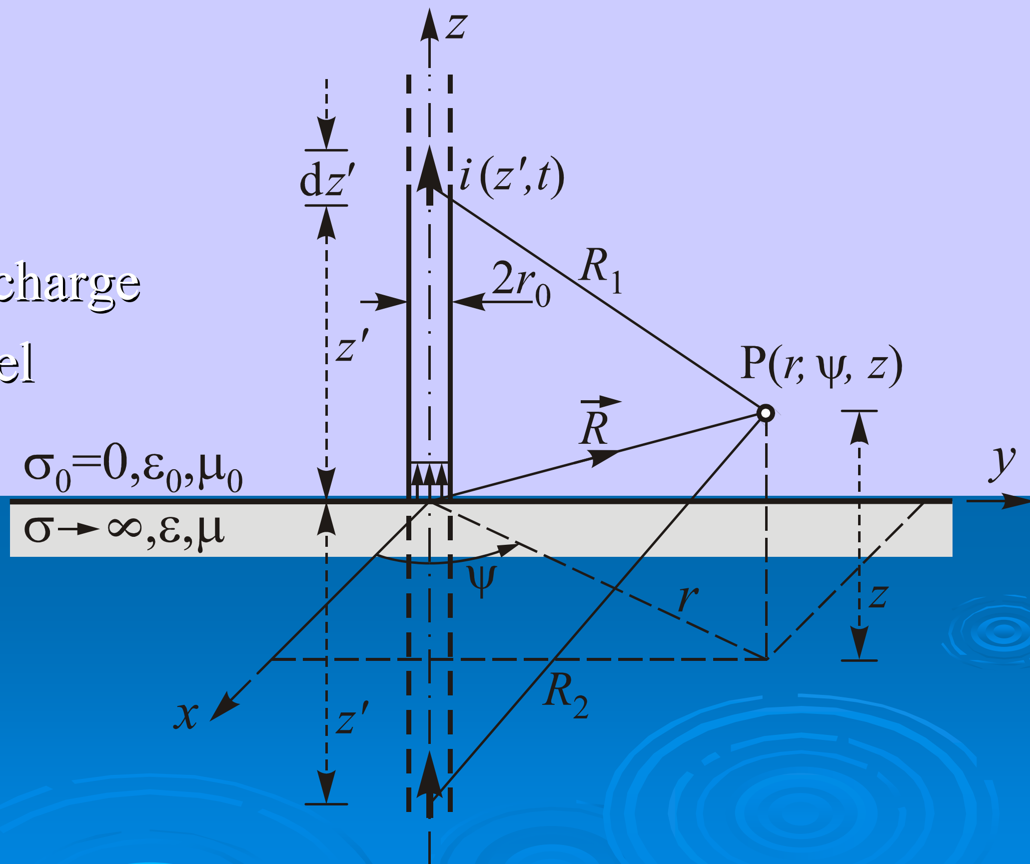


Fig.1 Lightning discharge antenna model

$r_0 << H$

New return stroke channel-base current (CBC)

Engineering models specify relation :

$$i(z',t) = u(t - z'/v_f)P(z',t)\,i(0,t - z'/v),$$

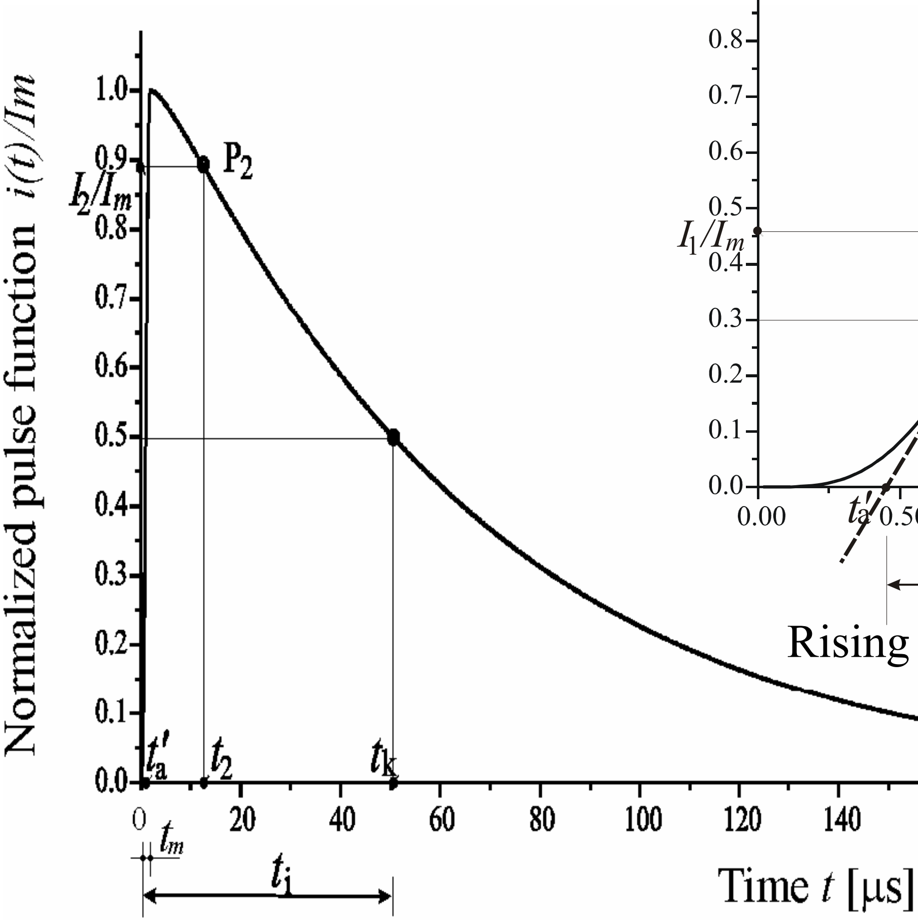
where: $u(t)-$ the Heaviside function, $v_f -$ the return-stroke speed, $v-$ the current-wave propagation speed and $P(z',t)-$ the height- and time-dependent current attenuation factor.

For MTLE model attenuation factor $P(z') = \exp(-z'/\lambda)$ is assumed, as only height-dependent, where λ is the adaptive constant and z' the height from the channel base.

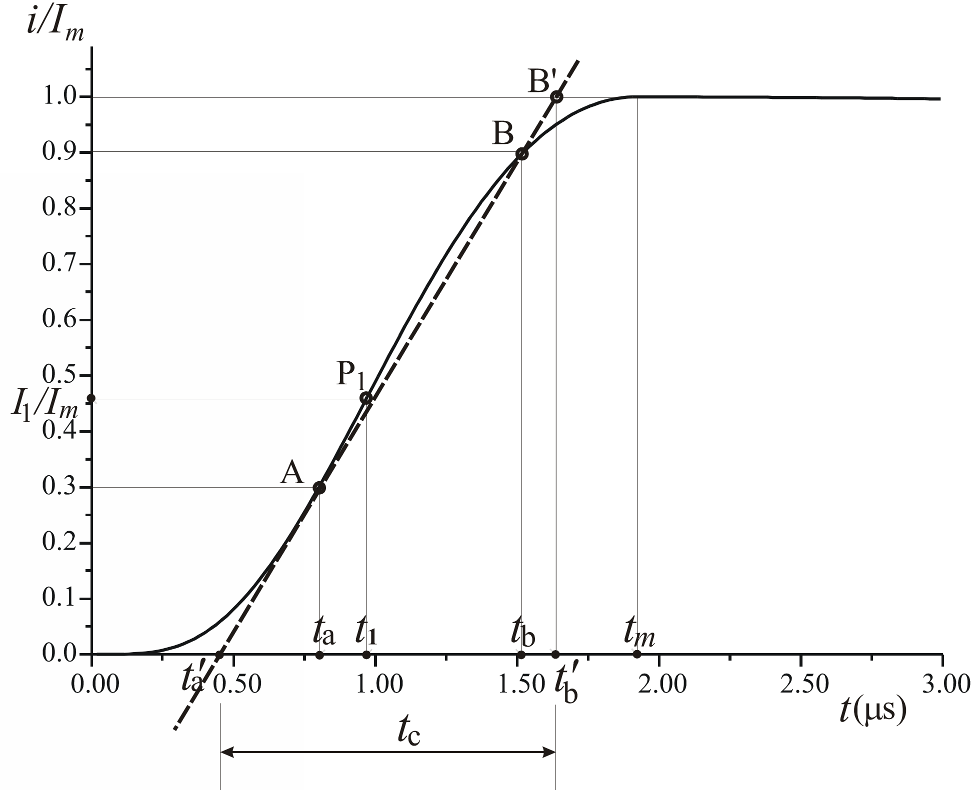
One new function for the return stroke lightning channel-base current (CBC) is proposed:

$$I(t) = \frac{i(t)}{I_m} = \begin{cases} \left[\tau\, \mathrm{e}^{(1-\tau)}\right]^a, & 0 \le \tau \le 1 \\ \left[\tau\, \mathrm{e}^{(1-\tau)}\right]^b, & 1 \le \tau < \infty \end{cases}$$

where: a and b– the parameters of the function, $\tau = t/t_m$, $t_m -$ rise-time to peak value and $I_m -$ the maximum current value.



Normalized CBC function versus time t.



Rising part of the normalized pulse function

Parameters of the CBC function

Double-exponential function proposed by Bruce and Golde (BG): $i(t) = I_m(\mathrm{e}^{-\alpha t} - \mathrm{e}^{-\beta t})$

$\alpha = 3 \cdot 10^4\,\mathrm{s}^{-1} \quad \beta = 10^7\,\mathrm{s}^{-1} \quad I_m = 11\,\mathrm{kA} \quad \Rightarrow \quad t_m = 0.5826\,\mu\mathrm{s} \quad t_k \cong 23\,\mu\mathrm{s}$

CBC function: a=0.5 and b=0.019, with $t_m = 0.5826\,\mu\mathrm{s} \quad I_m = 11\,\mathrm{kA} \quad \Rightarrow \quad t_k \cong 23\,\mu\mathrm{s}$

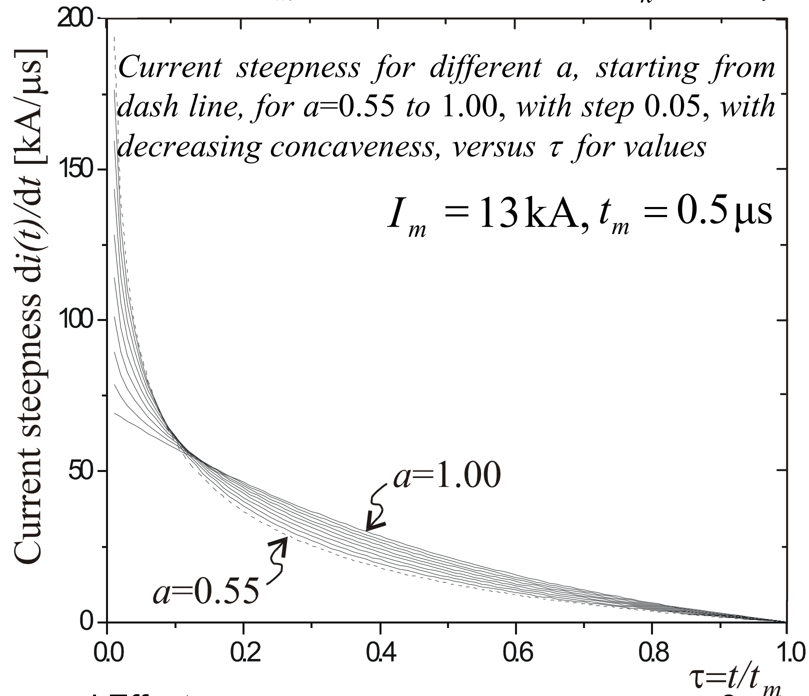
The first derivative of the CBC function

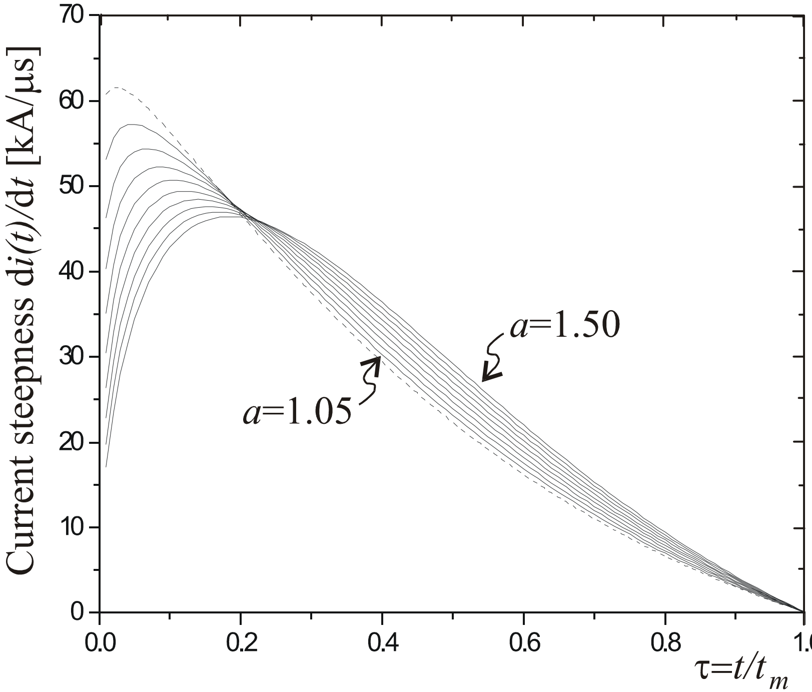
$$\mathrm{d}i/\mathrm{d}t = I_m\,\mathrm{d}I/\mathrm{d}t = I_m t_m^{-1}\,\mathrm{d}I/\mathrm{d}\tau$$

Parameter *a* determines current steepness in the rising part.

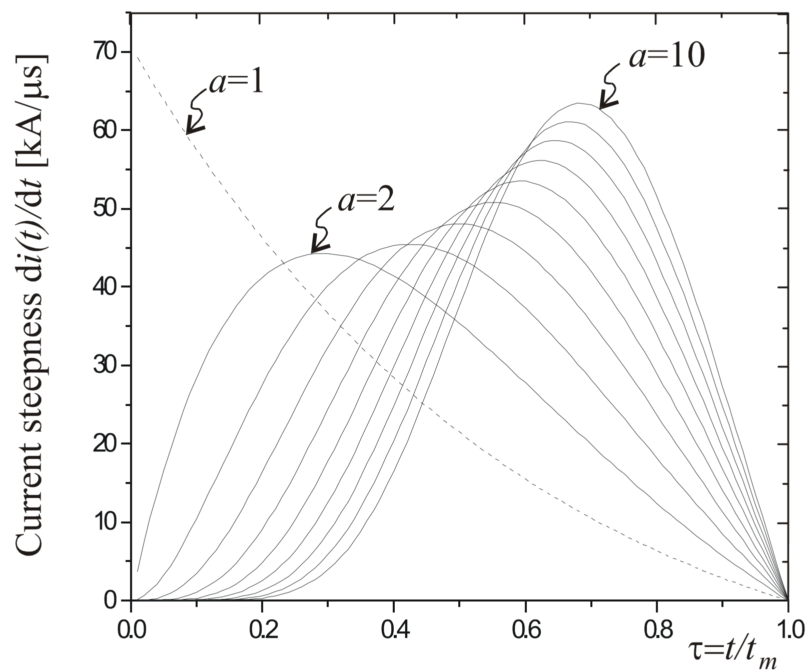
The rising part of the function can have

- **convex** shape without saddle point for $0 < a < 1$, or
- **concave to convex shape** and the saddle point in the rising part $a > 1$.

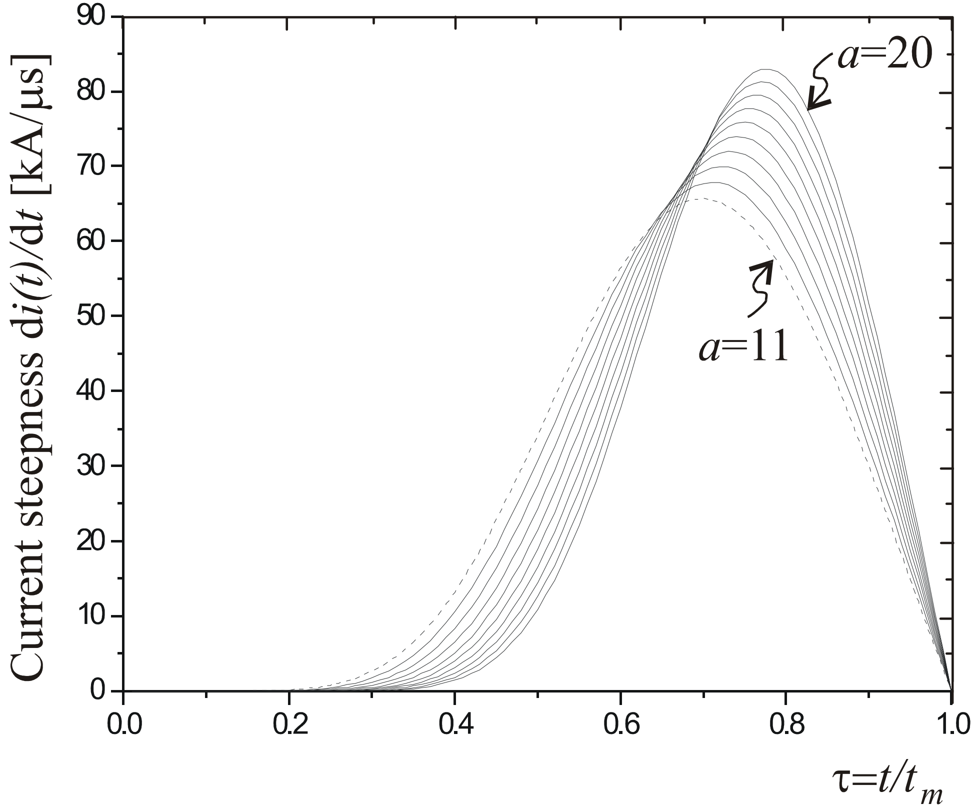




Current steepness for different values of a, starting From the dash line, for a=1.05 *to* 1.50, *with step* 0.05, *with decreasing maximum for increasing values of* τ .



Current steepness for different values of a, starting from a=1 *to* 10, *with step* 1, *from the dash line, with increasing maximum for increasing values of* τ .



Current steepness for different values of a starting from dash line, for a=11 *to* 20, *with step* 1, *with increasing maximum for increasing values of* τ .

Parameter *b*

It determines the shape of the decreasing part of the function
and thus the decreasing time to half of the peak value.
It mostly determines charge transfer of the return stroke current at the channel base.

For the proposed CBC function charge transfer is calculated analytically as the following:

$$q=\int_0^t i(t)\,\mathrm{d}t=\begin{cases}\dfrac{Q_0 e^a}{a^{a+1}}\gamma\left(a+1,a\dfrac{t}{t_m}\right), & 0\le t\le t_m,\\[2ex] \dfrac{Q_0 e^a}{a^{a+1}}\gamma(a+1,a)+ \\ +\dfrac{Q_0 e^b}{b^{b+1}}\left[\gamma\left(b+1,b\dfrac{t}{t_m}\right)-\gamma(b+1,b)\right], & t_m\le t\le\infty,\end{cases}$$

where: $Q_0 = I_m t_m$
and
$\gamma(a+1,x)=\int_0^x t^a\, e^{-t}\,\mathrm{d}t$ - the incomplete γ–function (Euler function of the second kind).

For chosen parameter a, parameter b can be determined so to obtain desired charge transfer.
Calculations are done by program Mathematica, but also using Fortran program.

With the CBC function approximate values of charges transferred up to $t_m = 0.5\mu s$, for $I_{max} = 13\,kA$, are calculated as following:

$$q_1 = 4.78\,mC \text{ for } a_1 = 0.9,$$
$$q_2 = 4.84\,mC \text{ for } a_2 = 0.85,$$
$$q_3 = 5.03\,mC \text{ for } a_3 = 0.7$$
$$\text{and} \quad q_4 = 5.25\,mC \text{ for } a_4 = 0.55.$$

The value of b must be determined, for corresponding value of a, such that the total charge transfer is $q = 50\,mC$ for $t \to \infty$, but practically already for $t = 100\,t_m$.

If calculated for each example, the following values are obtained for parameter b:

$$a_1 = 0.9 \qquad b_1 = 0.1953$$
$$a_2 = 0.85 \qquad b_2 = 0.1956$$
$$a_3 = 0.7 \qquad b_3 = 0.1967$$
$$a_4 = 0.55 \qquad b_4 = 0.1979.$$

for $I_{max} = 13\,\mathrm{kA}$ and $t_m = 0.5\,\mu\mathrm{s}$ $\Rightarrow$ $Q_0 = 6.5\,\mathrm{mC}$

Q_{uk} (mC)	*b*=0.03	*b*=0.02	*b*=0.01	*b*=0.001	*b*=0.0001	*b*=0.00001
a=0.5	242.89	353.45	682.44	6546.67	65061.5	650076.
a=1.0	242.22	352.78	681.77	6546.00	65060.8	650076.
a=1.5	241.76	352.33	681.32	6545.55	65060.4	650075.
a=2.0	241.43	352.00	680.99	6545.21	65060.0	650075.

for $I_{max} = 20\,\mathrm{kA}$ and $t_m = 1\,\mu\mathrm{s}$ $\Rightarrow$ $Q_0 = 20\,\mathrm{mC}$

Q_{uk} (C)	*b*=0.03	*b*=0.02	*b*=0.01	*b*=0.001	*b*=0.0001	*b*=0.00001
a=0.5	0.7474	1.0876	2.0999	20.144	200.194	2000.28
a=1.0	0.7453	1.0855	2.0978	20.142	200.192	2000.28
a=1.5	0.7439	1.0841	2.0964	20.141	200.191	2000.28
a=2.0	0.7429	1.0831	2.0954	20.140	200.190	2000.28

Characteristics of the new CBC function

The new function chosen for the lightning return stroke channel-base current is:

- continuous, as well as its first derivative;
- simple and integrable;
- can be convex or concave in the rising part;
- can have different desired current steepness;
- can give desired charge transfer;
- can have chosen rise-time to peak/increasing time;
- can have chosen time of decreasing to half of the peak value;
- can have chosen maximum value of the current, without peak correction factor.

There are also possibilities to adopt function parameters to satisfy other demands:

- values of the first and higher order derivatives which determine the function shape
- integral of the function itself or
- integral of the square of the function, etc.

  Disadvantage: the ratio τ determines the maximum of the current derivative for chosen concave to convex shape.

CALCULATING LEMF IN TIME DOMAIN

LEMF is numerically calculated directly in time domain using **antenna theory approach** and **thin wire approximation** of the lightning channel.

Lightning channel current is modelled using MTLE and **one new suitable function** for the approximation of the **return-stroke channel-base current**.

The function and its parameters are analyzed in order to obtain desired LEMP characteristics and to make proper choice of function parameters.

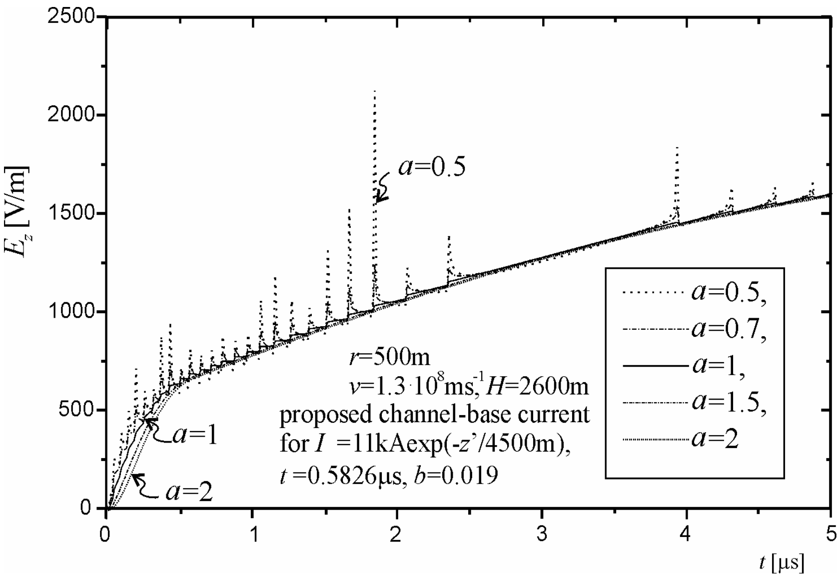
Electric and magnetic field components in cylindrical coordinates at the arbitrary field point

$$E_z(\vec{R},t) = \frac{1}{4\pi\varepsilon_0}\int_{-H}^{H}\left[\frac{2(z-z')^2 - r^2}{R_k^{\,5}}\int_{\tau=0}^{\tau=t} i\left(z',\tau-\frac{R_k}{c}\right)\mathrm{d}\tau + \frac{2(z-z')^2 - r^2}{c\,R_k^{\,4}}\, i\left(z',t-\frac{R_k}{c}\right) - \frac{r^2}{c^2 R_k^{\,3}}\frac{\partial i\left(z',t-\frac{R_k}{c}\right)}{\partial t}\right]\mathrm{d}z',$$

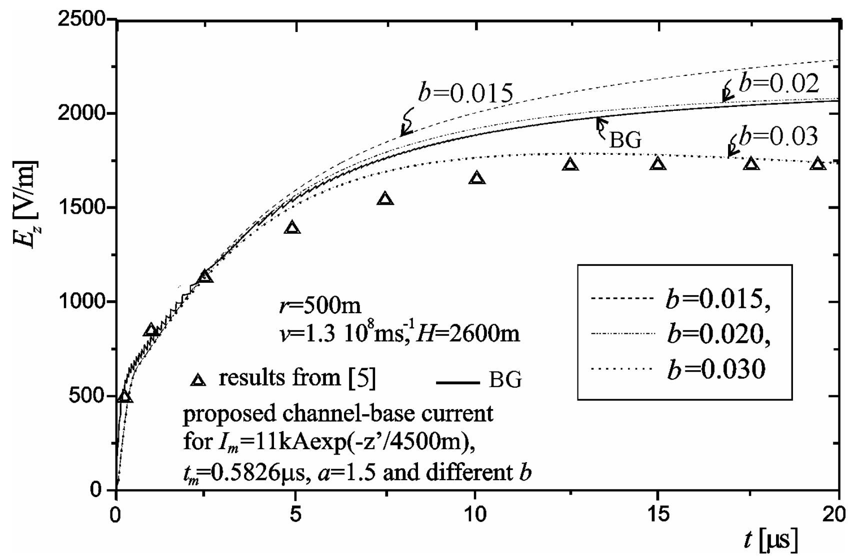
$$E_r(\vec{R},t) = \frac{1}{4\pi\varepsilon_0}\int_{-H}^{H}\left[\frac{3r(z-z')}{R_k^{\,5}}\int_{\tau=0}^{\tau=t} i\left(z',\tau-\frac{R_k}{c}\right)\mathrm{d}\tau + \frac{3r(z-z')}{c\,R_k^{\,4}}\, i\left(z',t-\frac{R_k}{c}\right) + \frac{r(z-z')}{c^2 R_k^{\,3}}\frac{\partial i\left(z',t-\frac{R_k}{c}\right)}{\partial t}\right]\mathrm{d}z',$$

$$H_\psi(\vec{R},t) = \frac{1}{4\pi}\int_{-H}^{H}\left[\frac{r}{R_k^{\,3}}\, i\left(z',t-\frac{R_k}{c}\right) + \frac{r}{c\,R_k^{\,2}}\frac{\partial i\left(z',t-\frac{R_k}{c}\right)}{\partial t}\right]\mathrm{d}z',$$

The influence of function parameters choice on the results for LEMF components



E_z *for* $r = 500\,\mathrm{m},\ z = 0$
for b=0.019 *and for different a.*



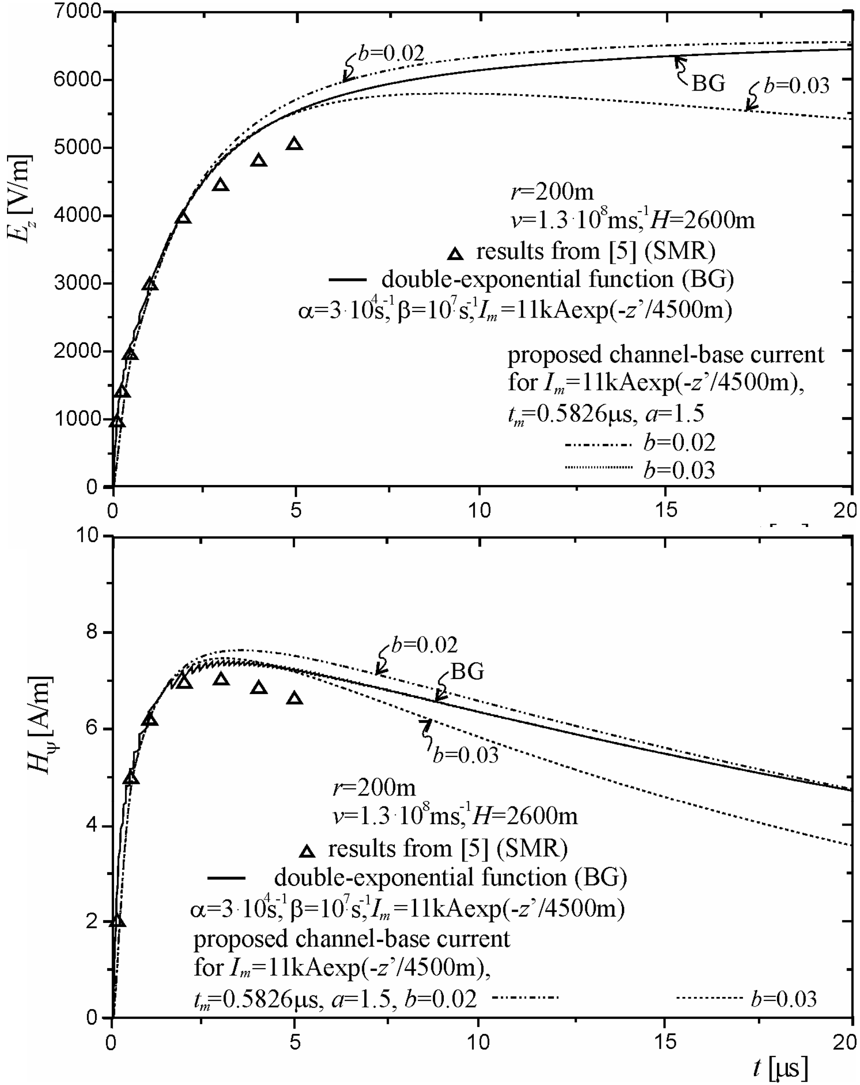
E_z *for* $r = 500\,\mathrm{m},\ z = 0$
for a=1.5 *and for different values of b.*

The results for LEMF components at different distances from the channel-base

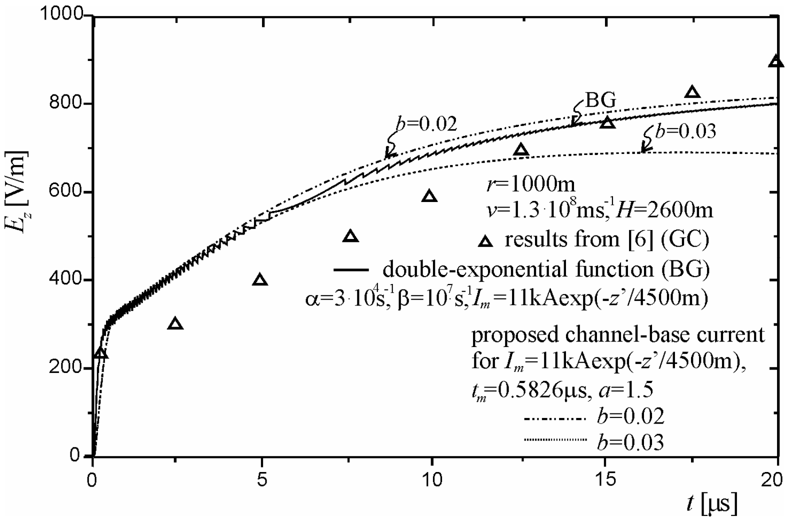
$$r_0 = 5\,\text{cm}$$
$$H = 2600\ \text{m}$$
$$v = 1.3 \cdot 10^8\ \text{ms}^{-1}$$

$$t_m = 0.5826\mu\text{s}$$
$$I_m = 11\text{kA}$$
$$P(z') = \exp(-z'/\lambda)$$
$$\lambda = 4500\ \text{m}$$

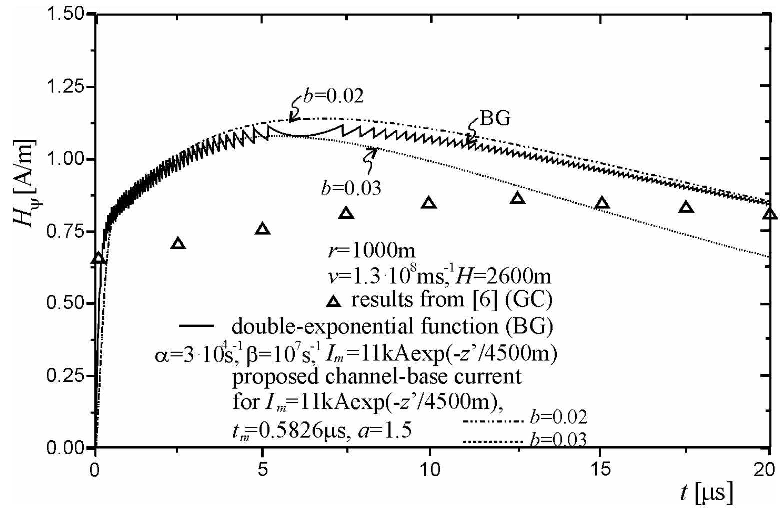
a=1.5, b=0.02 and b=0.03



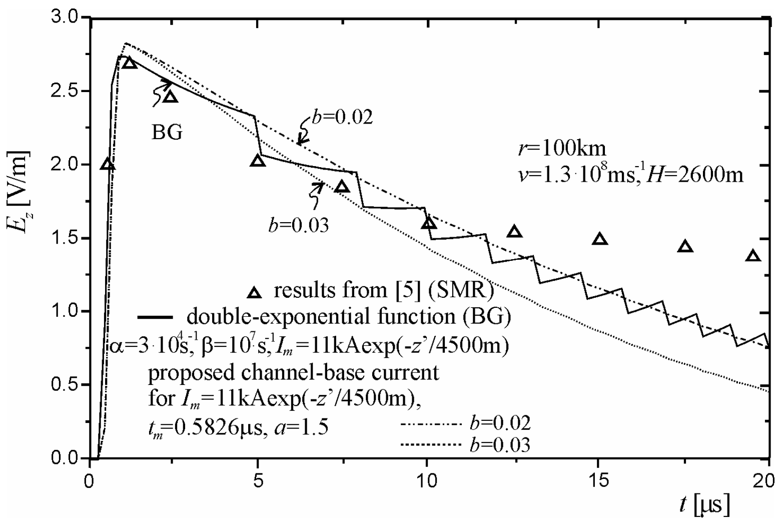
E_z | H_ψ



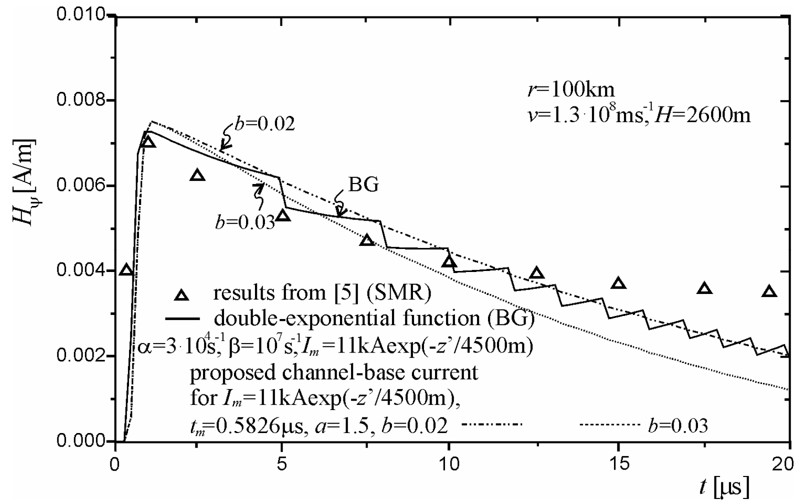
E_z [V/m]
t [µs]
b=0.02
BG
b=0.03
r=1000m
v=1.3·10^8 ms^-1, H=2600m
results from [6] (GC)
double-exponential function (BG)
α=3·10^4 s^-1, β=10^7 s^-1, I_m=11kAexp(-z'/4500m)
proposed channel-base current
for I_m=11kAexp(-z'/4500m),
t_m=0.5826µs, a=1.5
b=0.02
b=0.03



H_ψ [A/m]
t [µs]
b=0.02
BG
b=0.03
r=1000m
v=1.3·10^8 ms^-1, H=2600m
results from [6] (GC)
double-exponential function (BG)
α=3·10^4 s^-1, β=10^7 s^-1, I_m=11kAexp(-z'/4500m)
proposed channel-base current
for I_m=11kAexp(-z'/4500m),
t_m=0.5826µs, a=1.5
b=0.02
b=0.03



E_z [V/m]
t [µs]
BG
b=0.02
b=0.03
r=100km
v=1.3·10^8 ms^-1, H=2600m
results from [5] (SMR)
double-exponential function (BG)
α=3·10^4 s^-1, β=10^7 s^-1, I_m=11kAexp(-z'/4500m)
proposed channel-base current
for I_m=11kAexp(-z'/4500m),
t_m=0.5826µs, a=1.5
b=0.02
b=0.03

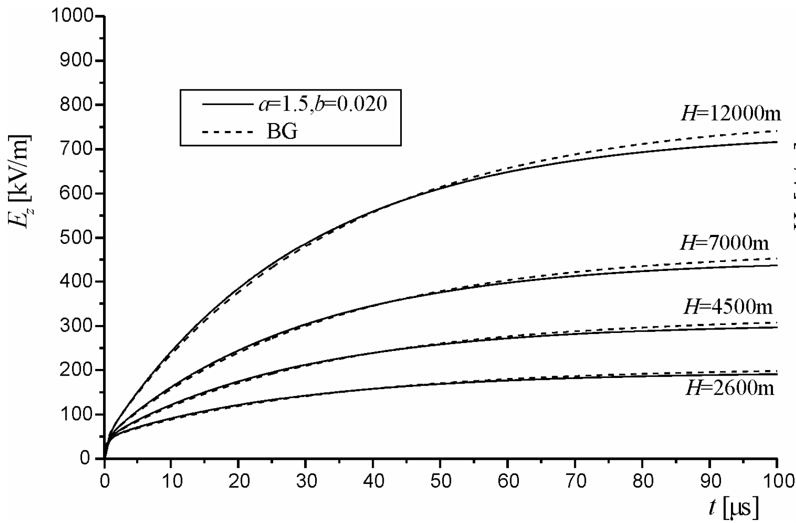


H_ψ [A/m]
t [µs]
r=100km
v=1.3·10^8 ms^-1, H=2600m
b=0.02
BG
b=0.03
results from [5] (SMR)
double-exponential function (BG)
α=3·10^4 s^-1, β=10^7 s^-1, I_m=11kAexp(-z'/4500m)
proposed channel-base current
for I_m=11kAexp(-z'/4500m),
t_m=0.5826µs, a=1.5, b=0.02
b=0.03

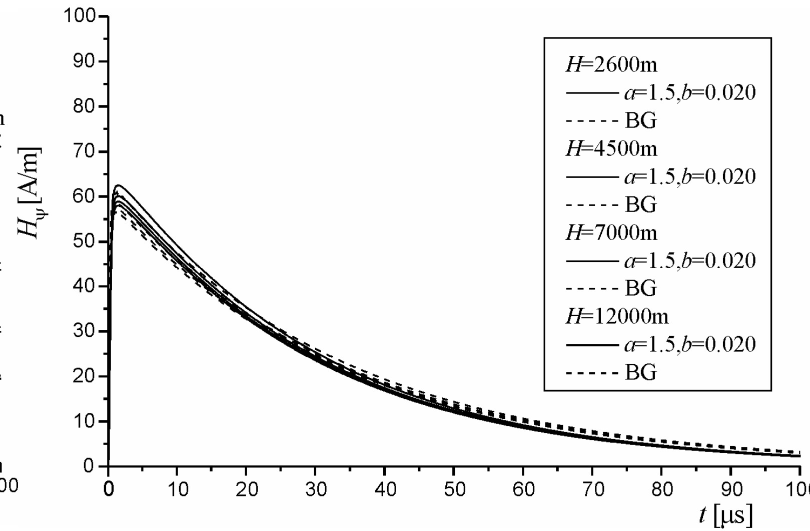
LIGHTNING CHANNEL HEIGHT INFLUENCE ON LEMF COMPONENTS

EF: In the near zone of the channel-base, at ground surface points, the influence is greater than in the far zone, where this height determines spike in a certain time moment after the onset of lightning electromagnetic pulse (LEMP).

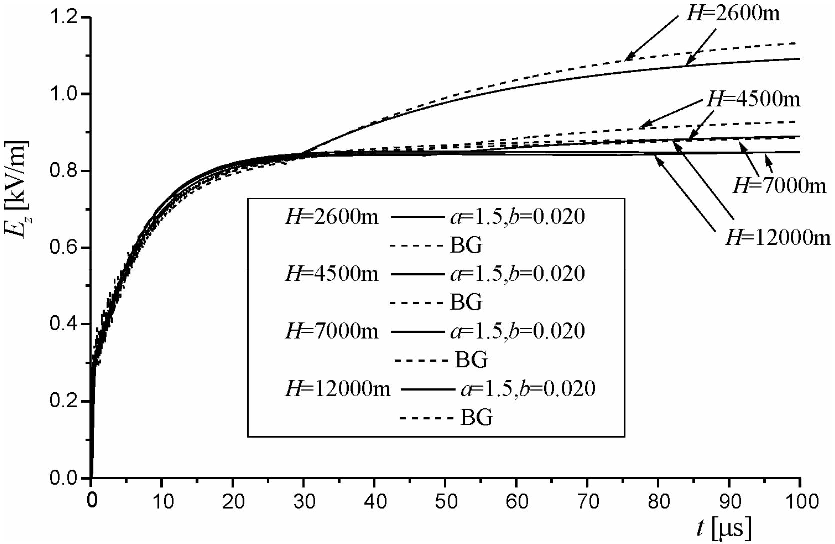
MF: Magnetic field for the points at ground surface does not differ a lot for different lightning channel heights for the near zone and for the far zone, but the choice of different channel heights also produces spikes in different time moments, as for EF.



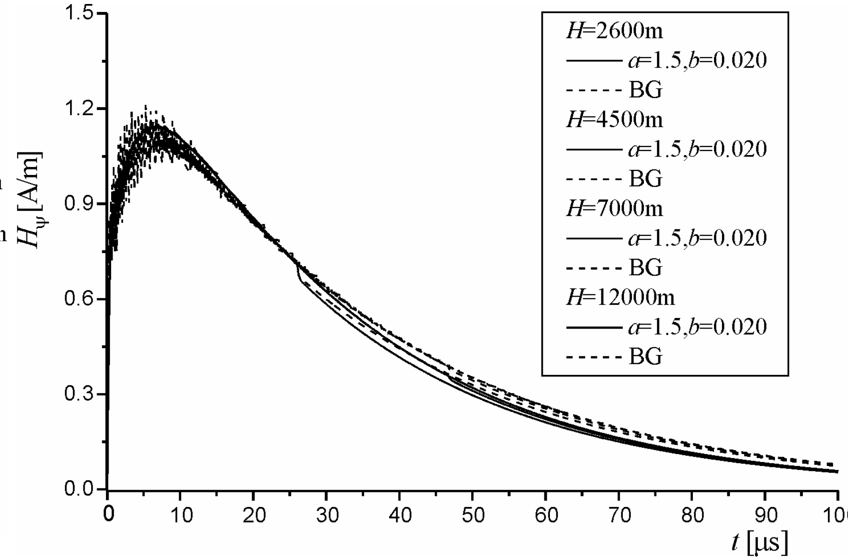
Electric field component for r=30m *and* z=0.



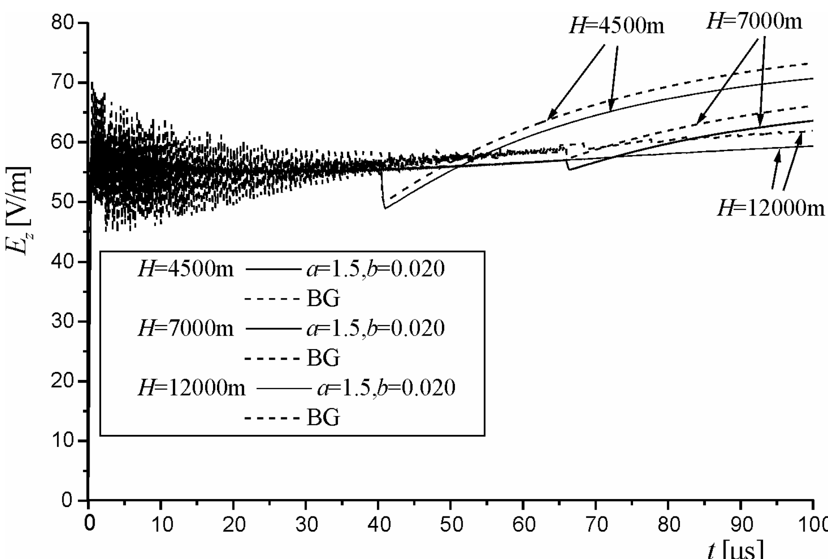
Magnetic field component for r=30m *and* z=0.



Electric field component for r=1km *and z*=0.

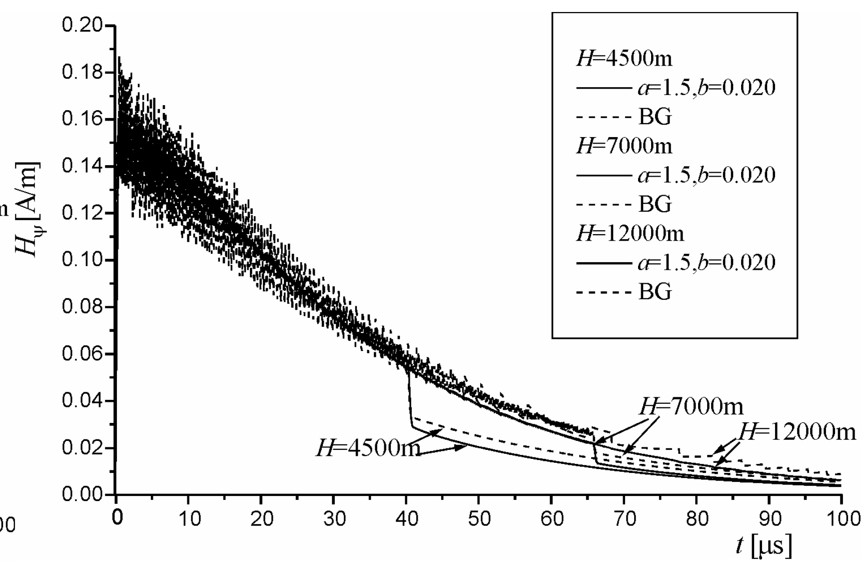


Magnetic field component for r=1km *and z*=0.



Electric field component

for r=5km *and z*=0.



Magnetic field component

for r=5km *and z*=0.

CONCLUSIONS

- Antenna theory approach and thin wire approximation of lightning channel are used for determining LEMF in time domain.
- Proposed function for the return stroke channel-base current gives satisfactory results for calculating lightning electromagnetic field.
- Parameters of this function can be chosen so that it satisfies some of the desired characteristics of the return stroke channel-base current and LEMP.
- The results for LEMF components at the field points at perfectly conducting ground surface for different distances from the lightning channel-base are in good agreement with the results from literature.
- Chosen simple return stroke channel-base function and determining LEMF in time domain provide easy analysis of interrupting vertical lightning channel on a certain height from its base.

REFERENCES

[1] V. A. Rakov, M. A. Uman, *Review and evaluation of lightning return stroke models including some aspects of their application*, IEEE Transactions on Electromagnetic Compatibility, Volume 40, No.4, November 1998, pp.403-426.

[2] Y. Baba, S. Miyazaki, M. Ishii, *Reproduction of lightning electromagnetic field waveforms by engineering model of return stroke*, IEEE Transactions on Electromagnetic Compatibility, Volume 46, No.1, February 2004, pp.130-133.

[3] D. Velickovic, S. Aleksic, *A new approximation of pulse phenomenon*, 19th ICLP, (distributed at the Conference), 1988, Graz, Austria.

[4] V. Javor, *Calculation of Lightning Electromagnetic Field in Time Domain: A New Return-Stroke Current Model*, Proc. of the 7th International Conference on Applied Electromagnetics PES'05, May 2005, Nis, Serbia, pp.259-268.

[5] A. Shoory, R. Moini, H. Sadeghi, V. A. Rakov, *Analysis of lightning-radiated electromagnetic fields in the vicinity of lossy ground*, IEEE Transactions on Electromagnetic Compatibility, Volume 47, No.1, February 2005, pp.131-145.

[6] M. J. Master, M. A. Uman, *Transient electric and magnetic fields associated with establishing a finite electrostatic dipole*, American J. Phys., Volume 51, No.2, February 1983, pp.118-126.

[7] Z. Feizhou, L. Shanghe, *A new function to represent the lightning return-stroke currents*, IEEE Transactions on Electromagnetic Compatibility, Volume 44, No.4, November 2002, pp.595-597.

[8] F. Heidler, J. M. Cvetic, B. V. Stanic, *Calculation of Lightning Current Parameters*, IEEE Trans. on Power Delivery, Vol.14, No.2, pp.399-404, 1999.

[9] V. Javor, P. D. Rancic, *Application of one suitable lightning return-stroke current model,* Proceedings of full papers, International Symposium on Electromagnetic Compatibility EMC Europe 2006, September 4-8, 2006, pp. 941-946, Barcelona, Spain, 2006.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