



COST P-18  
Fourth International Symposium  
on Lightning Physics and Effects

Vienna, May 27-29, 2009

# One Electromagnetic Model for Representing Lightning Attachment to the Lossy Ground

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# Outline

- **INTRODUCTION**
- **THEORETICAL APPROACH**
- **RESULTS**
- **CONCLUSION**

# INTRODUCTION

- For representing lightning attachment to the lossy ground in EM modeling of a lightning discharge channel (LDC) as a vertical mast antenna (VMA) a semi-spherical electrode is used.
- Real groundings of nearby lightning protection rods (LPRs) are also represented by the semi-spherical electrodes.
- The influence of these semi-spherical electrodes is taken into account in the governing equations of the EFIE type through the corresponding Hertzian dipoles.
- For the VMA the electrode is positioned immediately below the striking point and the groundings of lightning protection rods (LPRs) are presented as semi-spherical electrodes at their bases.
- The electrodes radii are equal to the radius of the lightning channel and to the LPRs cross-section radii.



# THEORETICAL APPROACH

The influence of grounding electrodes currents is modeled with the vertical Hertzian dipoles of the moments  $p_{gj} = I_j(0^+)a_{gj}$  for the current  $I_j(0^+)$  at the base of VMA/LPR having radius  $a_j$  for the grounding electrode radius  $a_{gj} = a_j$   $a_{gj} \ll \lambda_0$ ,  $j=1, \dots, J+1$  where  $J+1$  is the total number of antenna segments at the ground.

Hertzian dipoles can be included in EFIE e.g. System of integral equations of two potentials for the calculation of antenna currents and the resulting electromagnetic field.

$$\begin{aligned} & \underline{\gamma}_0 \int_{s=0}^{s_n} \varphi_0(s) \operatorname{ch}[\underline{\gamma}_0(s_n - s)] ds + \underline{\gamma}_0^2 \int_{s=0}^{s_n} \Pi_{s_n}(s) \operatorname{sh}[\underline{\gamma}_0(s_n - s)] ds \\ & + \int_{s=0}^{s_n} Z'_n(s) I_n(s) \operatorname{sh}[\underline{\gamma}_0(s_n - s)] ds = C_{2n} \operatorname{sh}(\underline{\gamma}_0 s_n) \end{aligned}$$

$$\varphi_0(s_n) + \underline{\gamma}_0 \int_{s=0}^{s_n} \varphi_0(s) \operatorname{sh}[\underline{\gamma}_0(s_n - s)] ds + \underline{\gamma}_0^2 \int_{s=0}^{s_n} \Pi_{s_n}(s) \operatorname{ch}[\underline{\gamma}_0(s_n - s)] ds + \int_{s=0}^{s_n} Z'_n(s) I_n(s) \operatorname{ch}[\underline{\gamma}_0(s_n - s)] ds = C_{2n} \operatorname{ch}(\underline{\gamma}_0 s_n)$$

$\varphi_0(s) = -\operatorname{div} \vec{\Pi}_0 = -\partial \Pi_{z0} / \partial z$  is the scalar potential and  $\Pi_{s_n}(s) = \Pi_{z0}(s)$  is the Hertz vector tangential component, both calculated at the  $n$ -th conductor surface.

$$\Pi_{z0}(s) = \frac{1}{4\pi\sigma_0} \sum_{k=1}^{N+N_J} \left\{ \int_{s'_k=0}^{l_k} I_k(s'_k) [K_0(r_{1k}) + S_{00}^v(r_{2k})] ds'_k + (\delta_{1k} + \sum_{j=1}^J \delta_{j+1,k}) p_{gk} S_{01}^v(r_{2k}, s'_k = 0) \right\} \Big|_{z=s}$$

$$\varphi_0(s) = \frac{1}{4\pi\sigma_0} \sum_{k=1}^{N+N_J} \left\{ \int_{s'_k=0}^{l_k} I_k(s'_k) \frac{\partial}{\partial s'_k} [K_0(r_{1k}) - S_{00}^v(r_{2k})] ds'_k - (\delta_{1k} + \sum_{j=1}^J \delta_{j+1,k}) p_{gk} \frac{\partial}{\partial z} S_{01}^v(r_{2k}, s'_k = 0) \right\} \Big|_{z=s}$$

$$I_k(s'_k) = \sum_{m=0}^{n_k} B_{mk} (s'_k / l_k)^m$$

# RESULTS

The results for the impedance of one semi-spherical grounding at the VMA base are obtained for different VMA heights as the functions of normalized ground conductivity for different electric permittivity of the ground.

The results are compared to the semi-spherical grounding impedance calculated from the quasistationary expression

$$\underline{Z}_G \cong (2\pi\underline{\sigma}_1 a_g)^{-1}$$

and presented for different ground parameters  $\varepsilon_1$  and  $\sigma_1$ .

## Input antenna impedance:

$$Z_A = R_A + jX_A = \frac{\varphi_0(s_1 = 0^+)}{I_1(s'_1 = 0)}$$

## Grounding impedance:

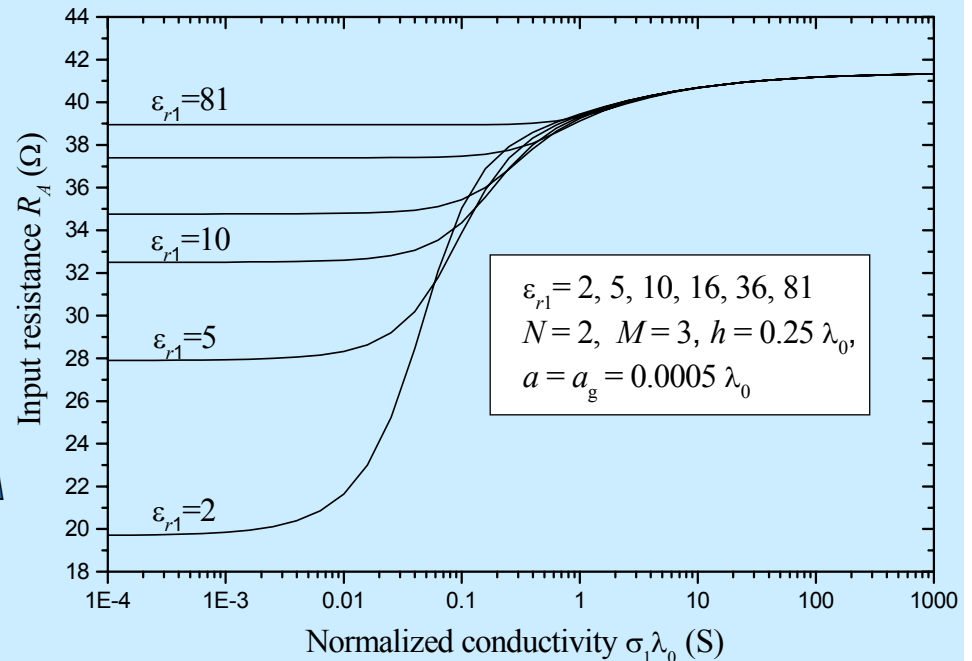
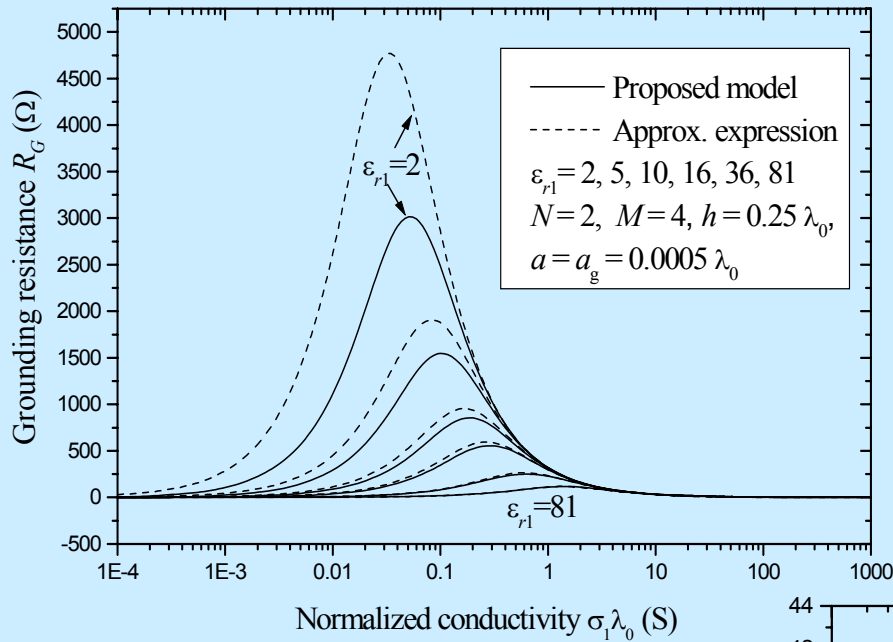
$$Z_G = R_G + jX_G = -\frac{\varphi_B}{I_1(s'_1 = 0)} = -Z_A \frac{\varphi_B}{\varphi_A}$$

## Input impedance of the system:

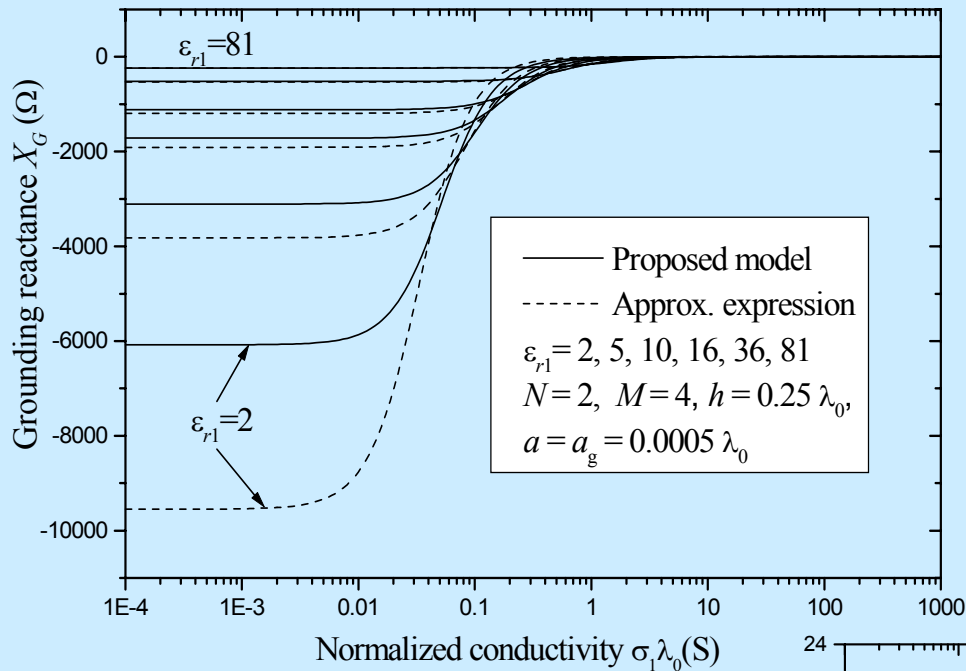
$$Z_{IN} = R_{IN} + jX_{IN} = Z_A + Z_G = \frac{U}{I_1(s'_1 = 0)} = Z_A \left(1 - \frac{\varphi_B}{\varphi_A}\right)$$

$$U = \varphi_A - \varphi_B$$

## Grounding resistance $R_G$ for $h = 0.25 \lambda_0$

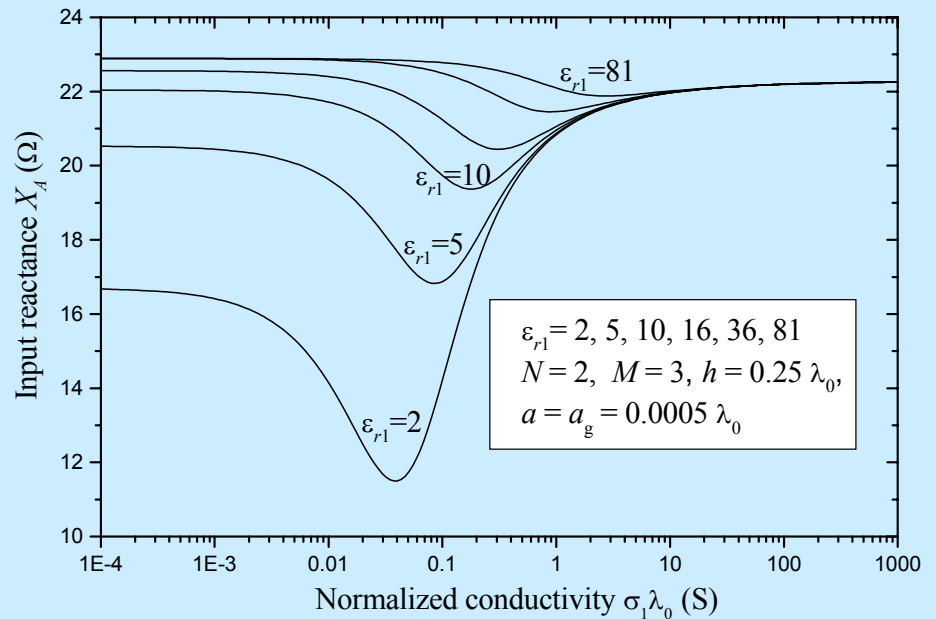


## Input antenna resistance $R_A$ for $h = 0.25 \lambda_0$

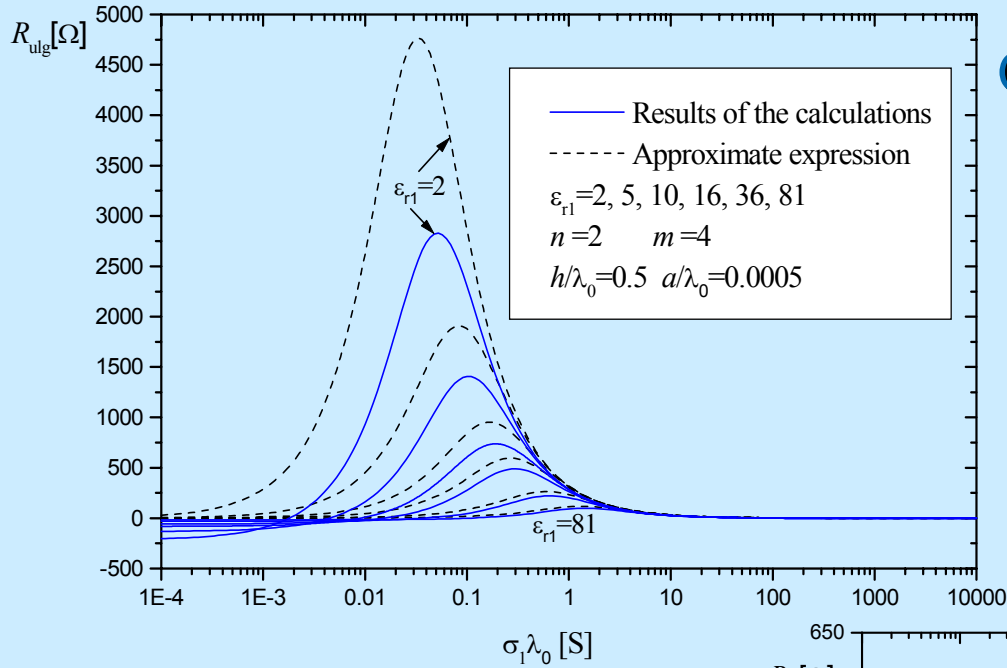


Grounding reactance  $X_G$   
for  $h = 0.25 \lambda_0$

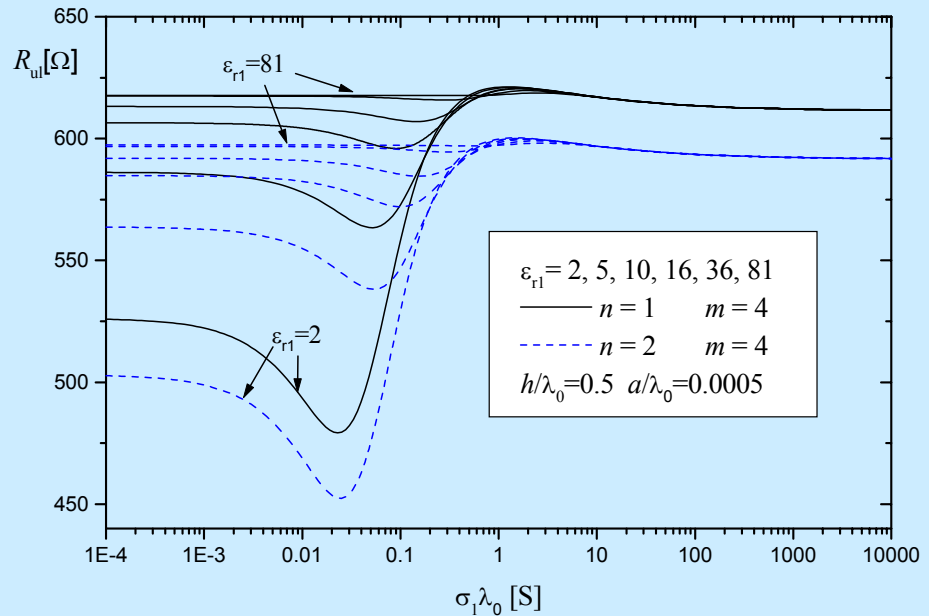
Input antenna reactance  $X_G$   
for  $h = 0.25 \lambda_0$

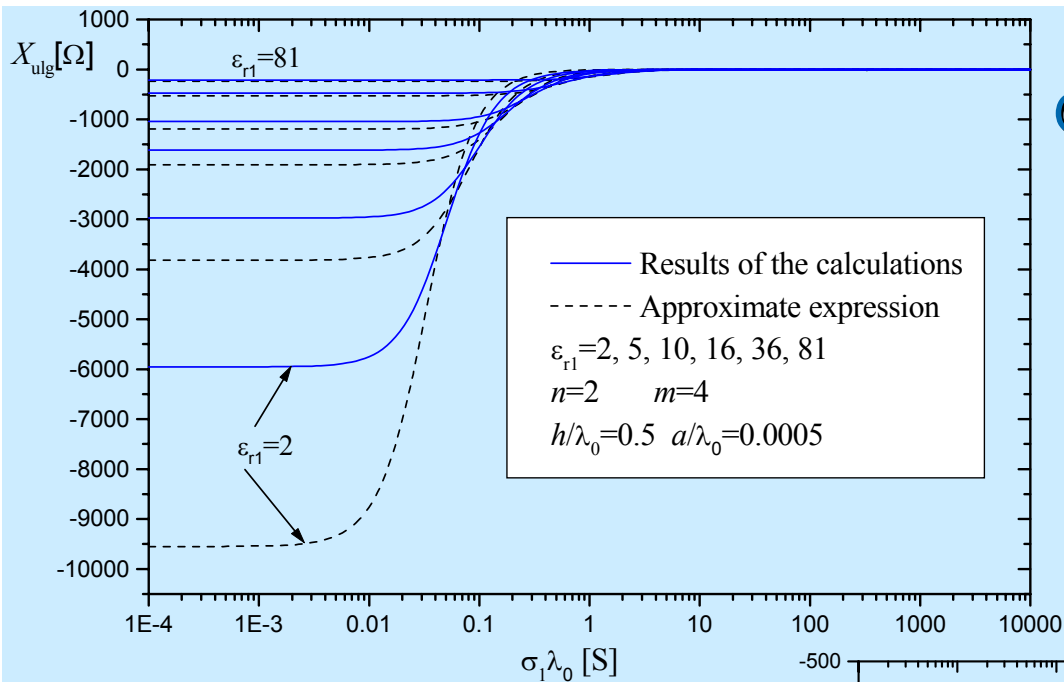


# Grounding resistance $R_G$ for $h = 0.50 \lambda_0$



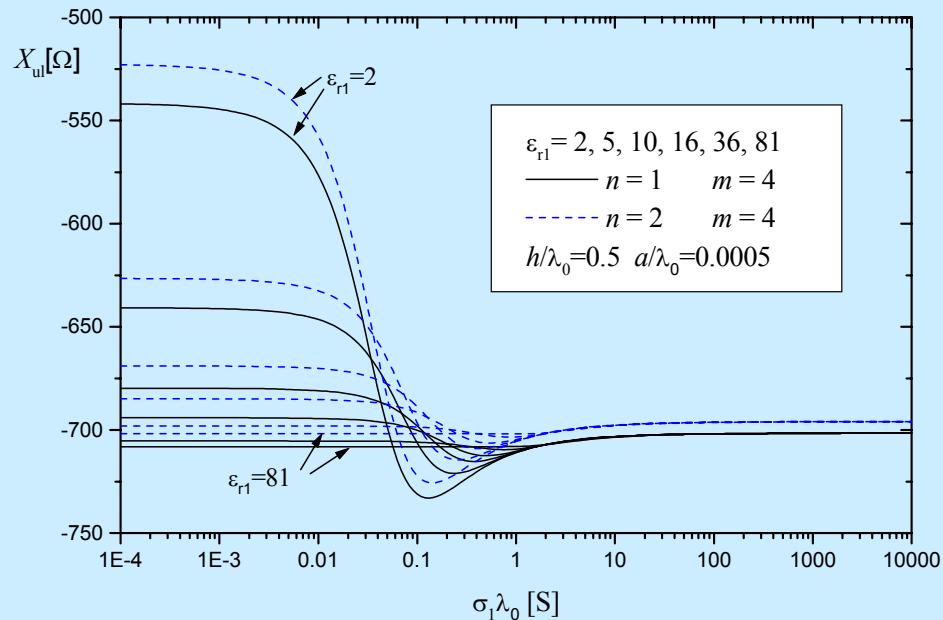
# Input antenna resistance $R_A$ for $h = 0.50 \lambda_0$

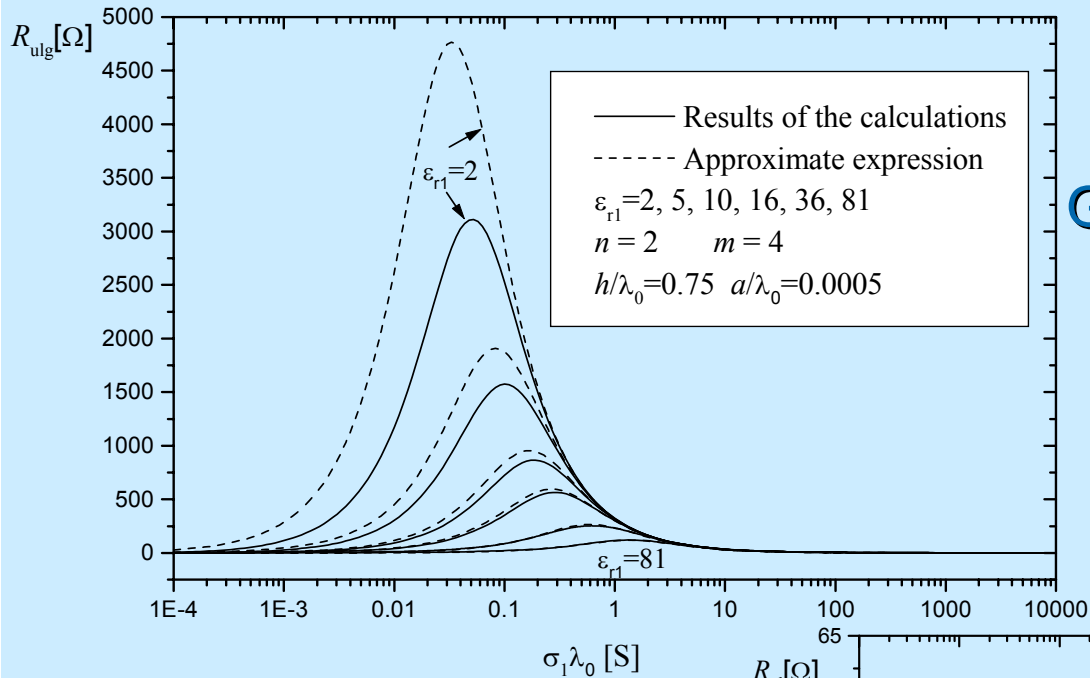




Grounding reactance  $X_G$   
for  $h = 0.50 \lambda_0$

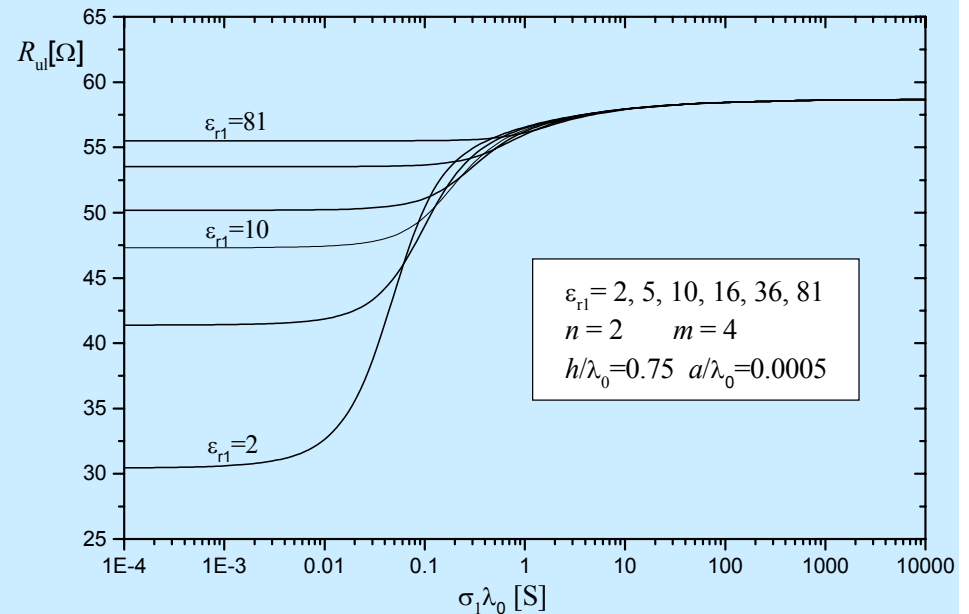
Input antenna reactance  $X_A$   
for  $h = 0.50 \lambda_0$

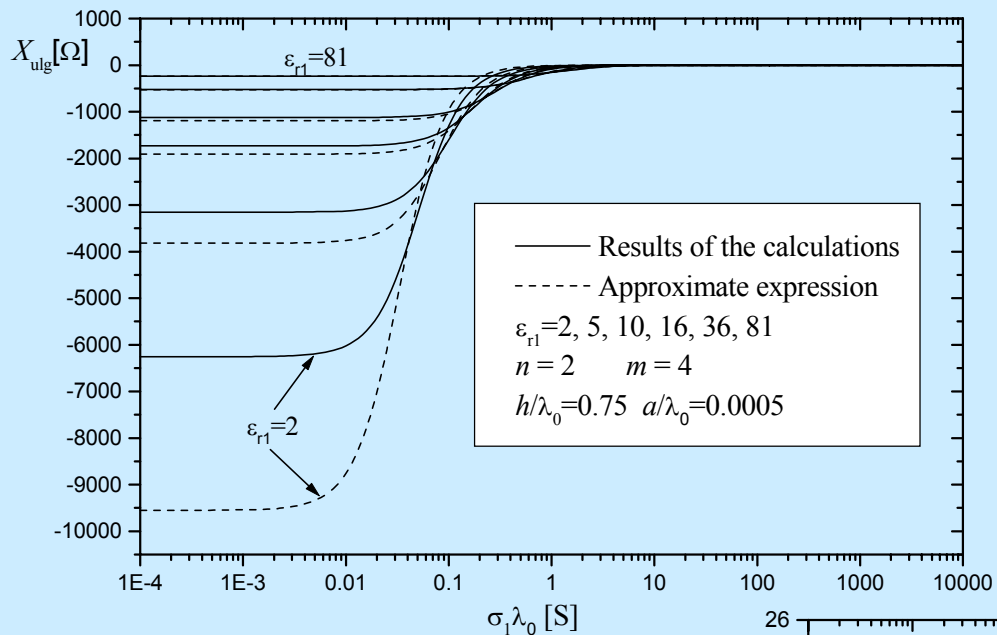




Grounding resistance  $R_G$   
 for  $h = 0.75 \lambda_0$

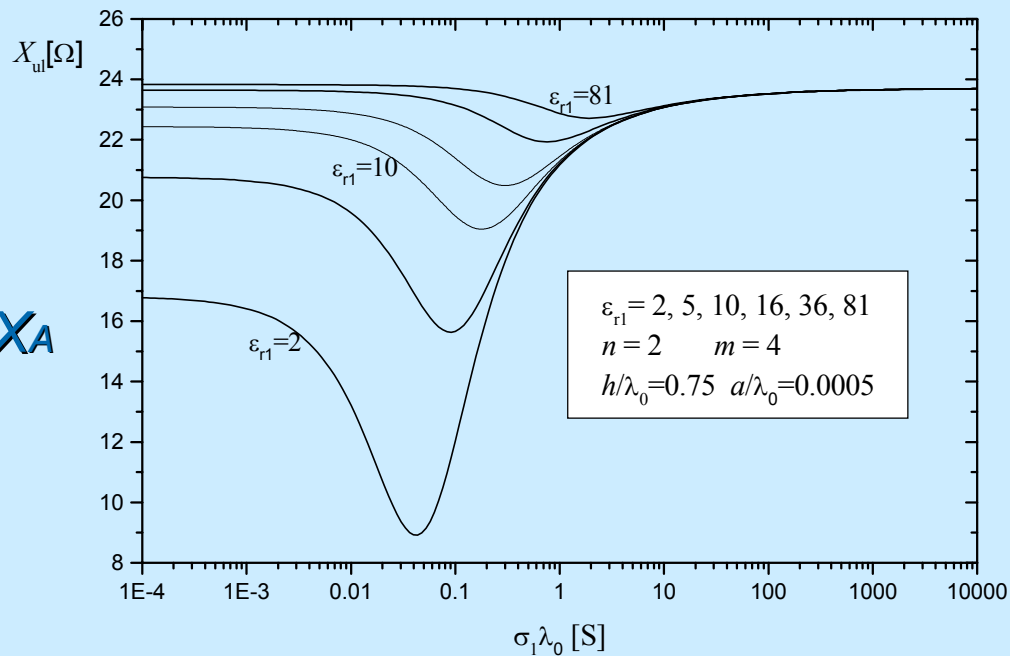
Input antenna resistance  $R_A$   
 for  $h = 0.75 \lambda_0$



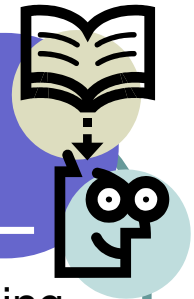


Grounding reactance  $X_G$   
for  $h = 0.75 \lambda_0$

Input antenna reactance  $X_A$   
for  $h = 0.75 \lambda_0$



# CONCLUSION



- An antenna approach and a thin-wire approximation of the lightning discharge channel is used in this paper.
- The lightning attachment point representation with a semi-spherical electrode is used and the influence of the vertical lightning protection rods groundings is included in the EFIE equations for obtaining current distributions along the VMA and LPRs at a lossy ground.
- Current distributions are approximated by polynomials and determined by using the System of integral equations of two potentials, numerically solved by the method of moments.
- The input impedance of a VMA-LPRs system is determined, so as the grounding impedance and the VMA antenna impedance. The results are obtained using self-made program SPAN and presented for different ground parameters and different antenna heights.



**THANK YOU  
FOR YOUR  
ATTENTION!**