



A Two-Station Lightning Location Method based on a Combination of Difference of Time of Arrival and Amplitude Attenuation

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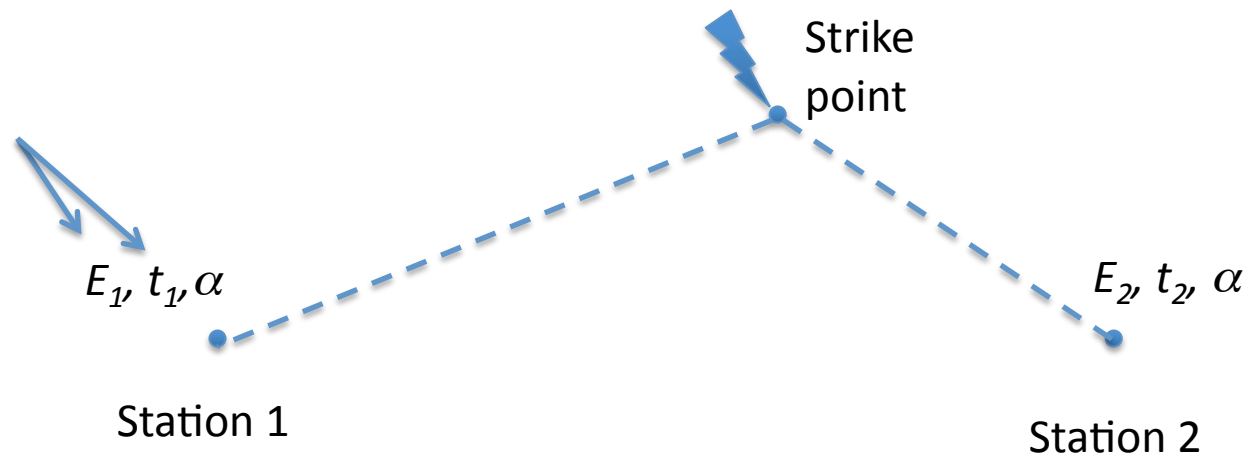
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Outline

- Electric Field Peak based
- Time-difference based
- Combined Method
- Some words about errors
- Advantages and disadvantages

The General Problem

- Assume two sensors



Electric Field Peak Amplitudes

- The ratio of the measured radiation electric field peaks is known:

$$E_{peak} = \frac{k}{D^\eta} \quad \frac{E_{peak1}}{E_{peak2}} = \left(\frac{D_2}{D_1} \right)^\eta = \gamma$$

- What does this ratio tell us about the location of the strike point?
- It lies in an Apollonius circle...

Electric Field

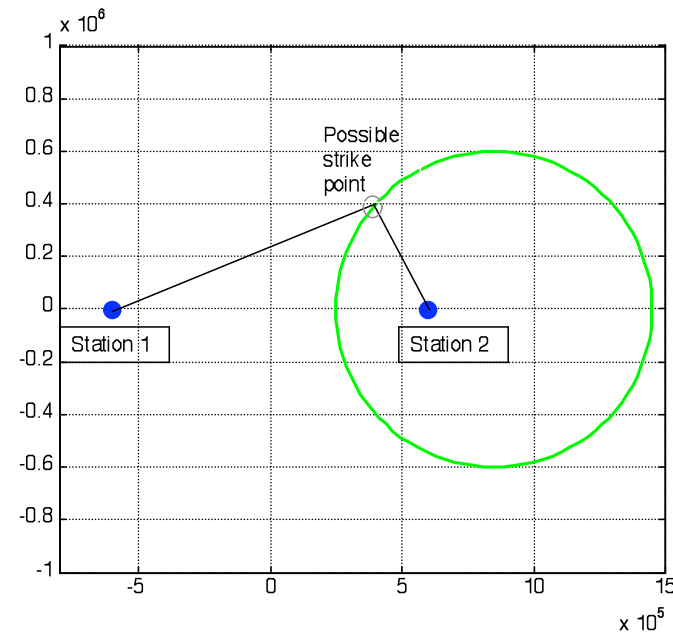
- Locus of points for which the ratio of the distances to two focal points is constant

$$(x + x_0)^2 + y^2 = \gamma^{2/\eta} \left((x - x_0)^2 + y^2 \right)$$

$$\left(x - x_0 \left(\frac{1 + \gamma^{2/\eta}}{\gamma^{2/\eta} - 1} \right) \right)^2 + y^2 = -x_0^2 + \left(x_0 \left(\frac{1 + \gamma^{2/\eta}}{\gamma^{2/\eta} - 1} \right) \right)^2$$

$$\text{center} = x_0 \left(\frac{1 + \gamma^{2/\eta}}{\gamma^{2/\eta} - 1} \right)$$

$$\text{radius} = \left| \frac{2x_0\gamma^{1/\eta}}{\gamma^{2/\eta} - 1} \right|$$



Difference of time of arrival

- Uses difference of time of arrival to the two sensors $ct_1 - ct_2 = D_1 - D_2$
- Can readily write the relation between x and y

$$\sqrt{(x - x_0)^2 + y^2} - \sqrt{(x + x_0)^2 + y^2} = c\Delta t$$

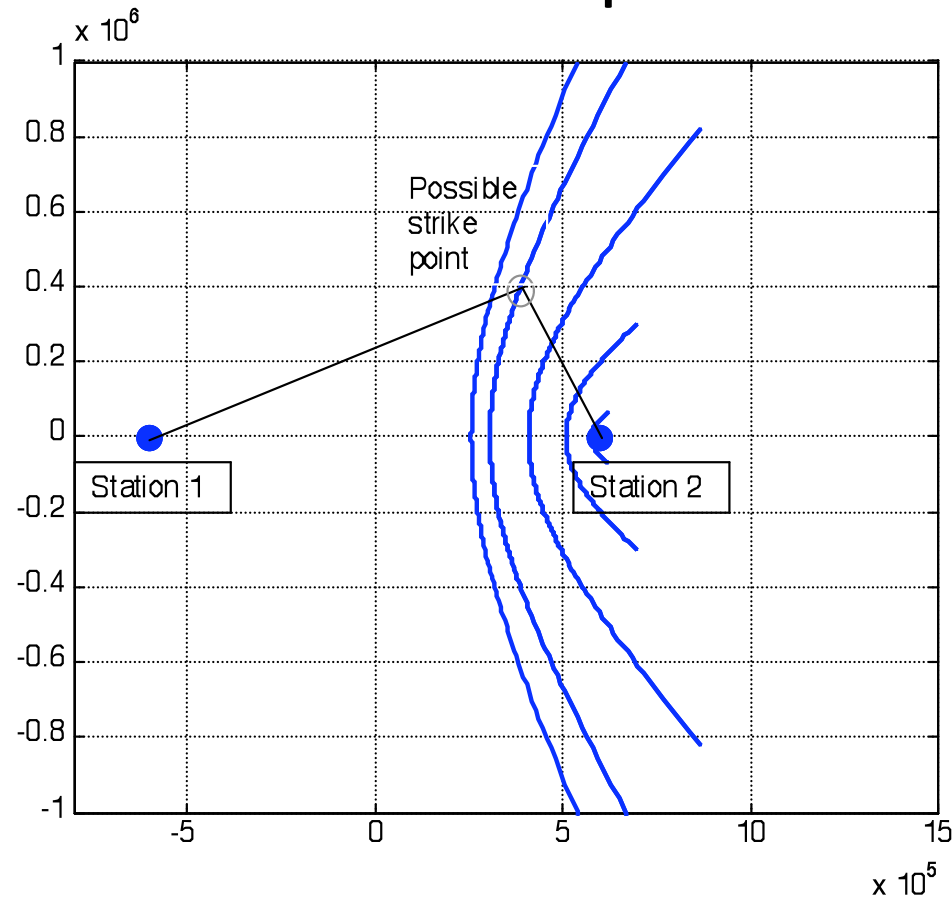
$$x^2 \left(x_0^2 - \left(\frac{c\Delta t}{2} \right)^2 \right) - y^2 \left(\left(\frac{c\Delta t}{2} \right)^2 \right) = \left(\frac{c\Delta t}{2} \right)^2 \left(x_0^2 - \left(\frac{c\Delta t}{2} \right)^2 \right)$$

$$\frac{x^2}{\left(\frac{c\Delta t}{2} \right)^2} - \frac{y^2}{b^2} = 1$$

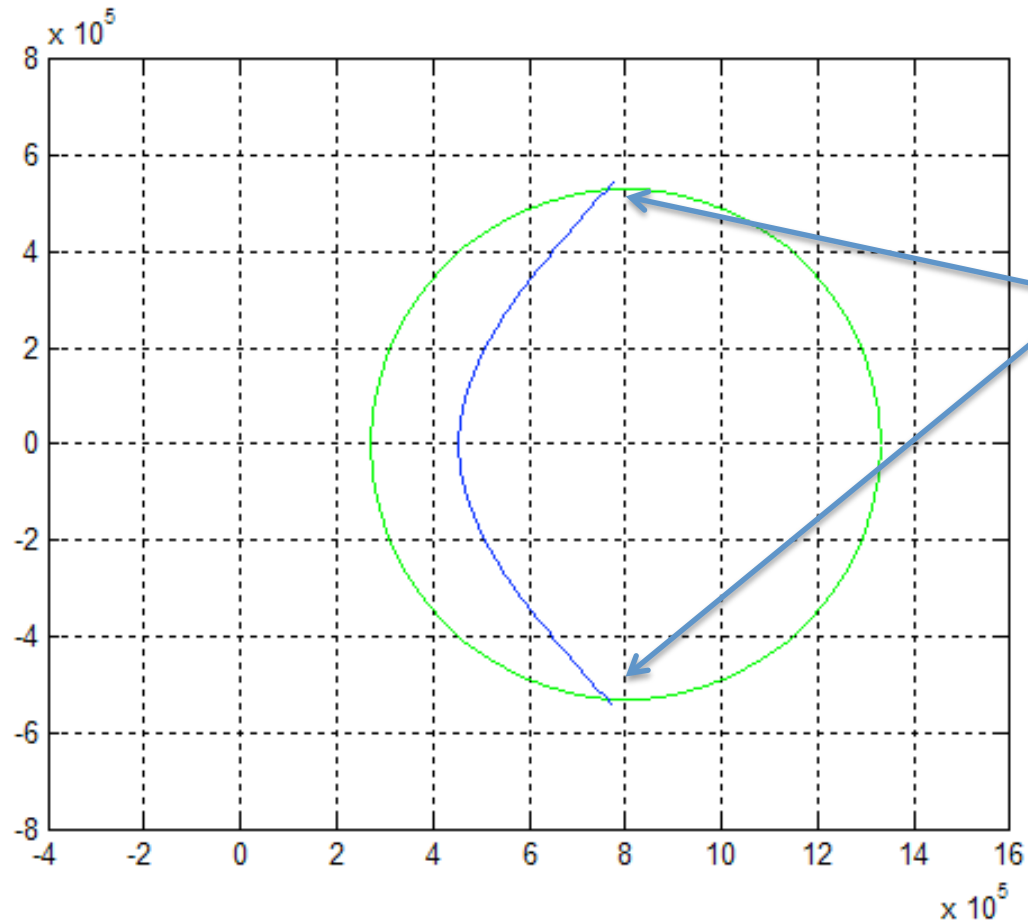
$$\frac{x^2}{\left(\frac{c\Delta t}{2} \right)^2} - \frac{y^2}{x_0^2 - \left(\frac{c\Delta t}{2} \right)^2} = 1$$

Difference of time of arrival

- Locus of the points with a constant difference in the distance to two focal points is a hyperbola



The intersection of the circle and the hyperbolic branch is the strike point



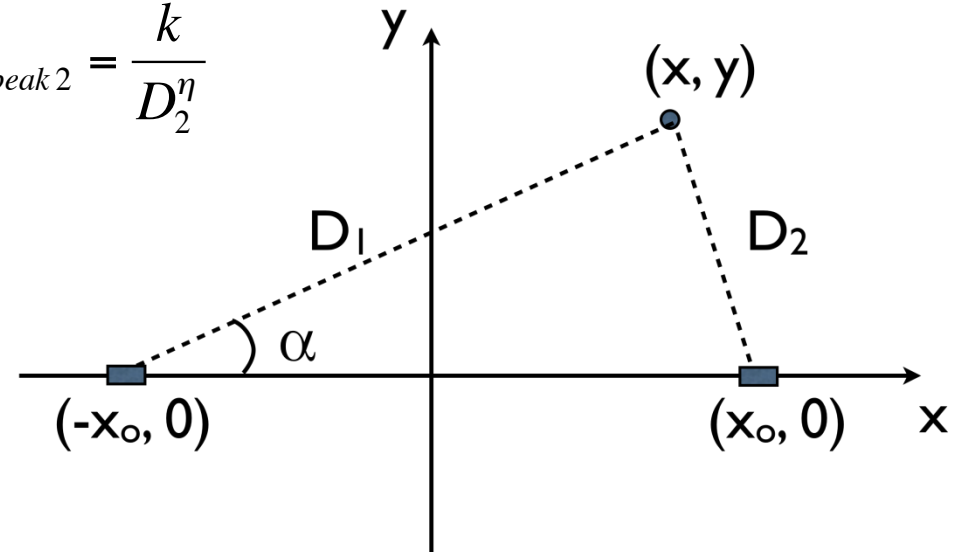
There is an ambiguity

Analytical equations

- Uses time-differences and peak Electric Field

$$\Delta t = D_1/c - D_2/c \quad E_{peak1} = \frac{k}{D_1^\eta} \quad E_{peak2} = \frac{k}{D_2^\eta}$$

$$D_2 = \frac{c\Delta t}{\left(\frac{E_{peak2}}{E_{peak1}}\right)^{1/\eta} - 1} \quad D_1 = \frac{c\Delta t}{1 - \left(\frac{E_{peak1}}{E_{peak2}}\right)^{1/\eta}}$$

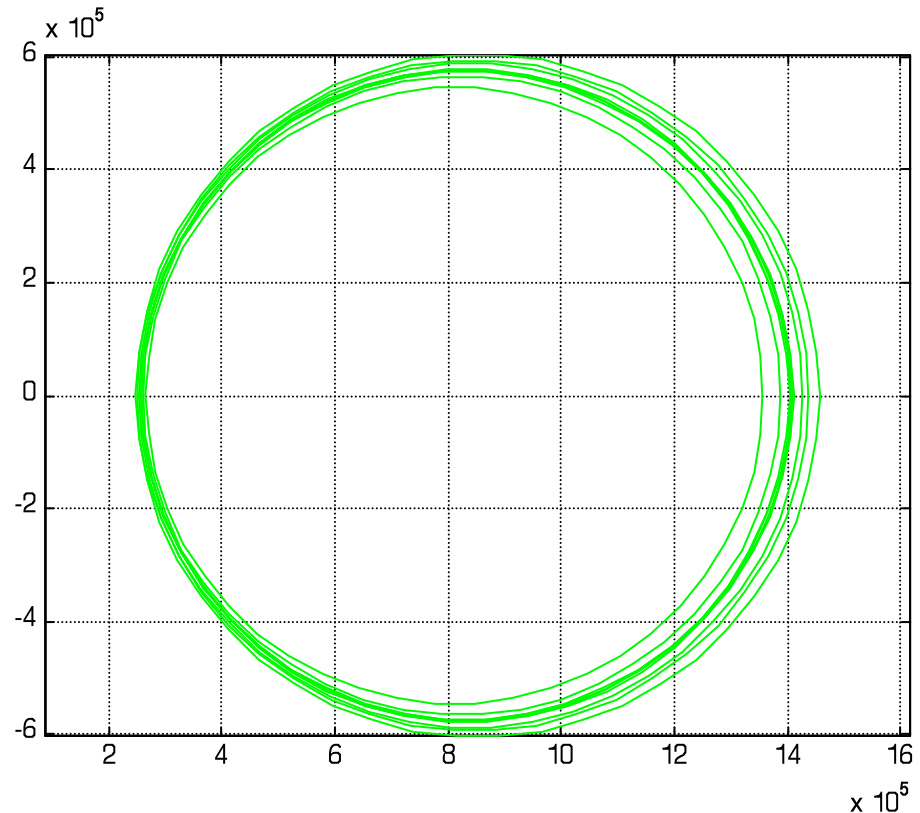


$$x = \frac{c^2 \Delta t^2 \frac{E_{peak2} + E_{peak1}}{E_{peak2} - E_{peak1}}}{4x_o} \quad y = \pm D_1 \sqrt{1 - \left(\frac{D_1^2 - D_2^2 + (2x_o)^2}{4D_1 x_o}\right)^2}$$

k is also found as an important byproduct of the location procedure

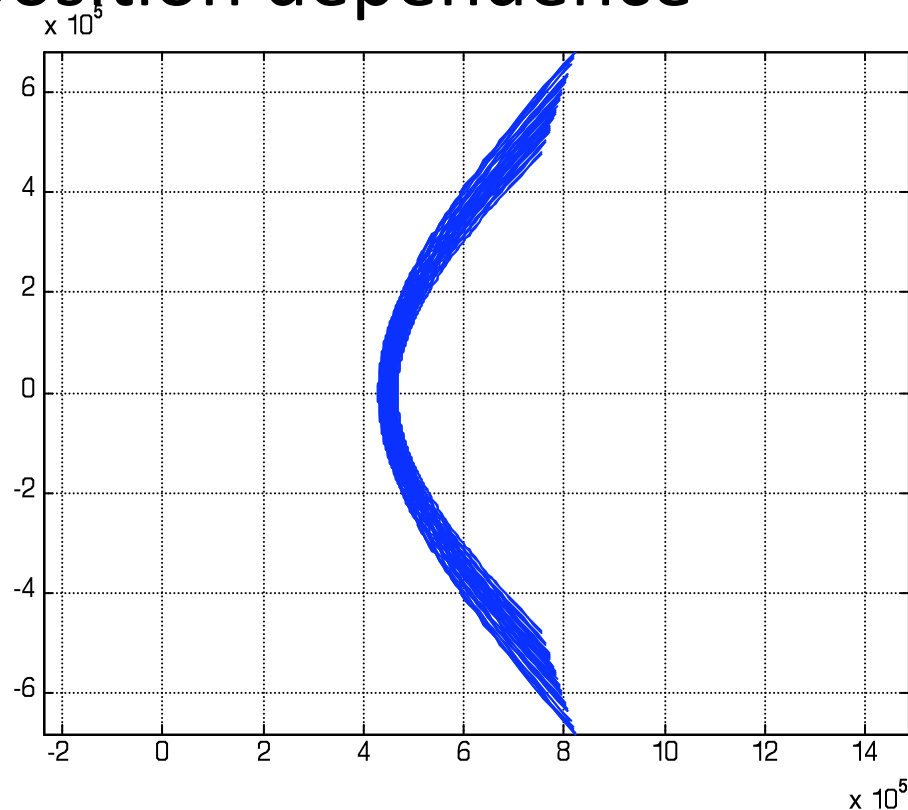
Monte Carlo Analysis for E Field

- Generate Random input vectors with known distribution, mean and variance
- Vectors with SD=5%
- Position dependent



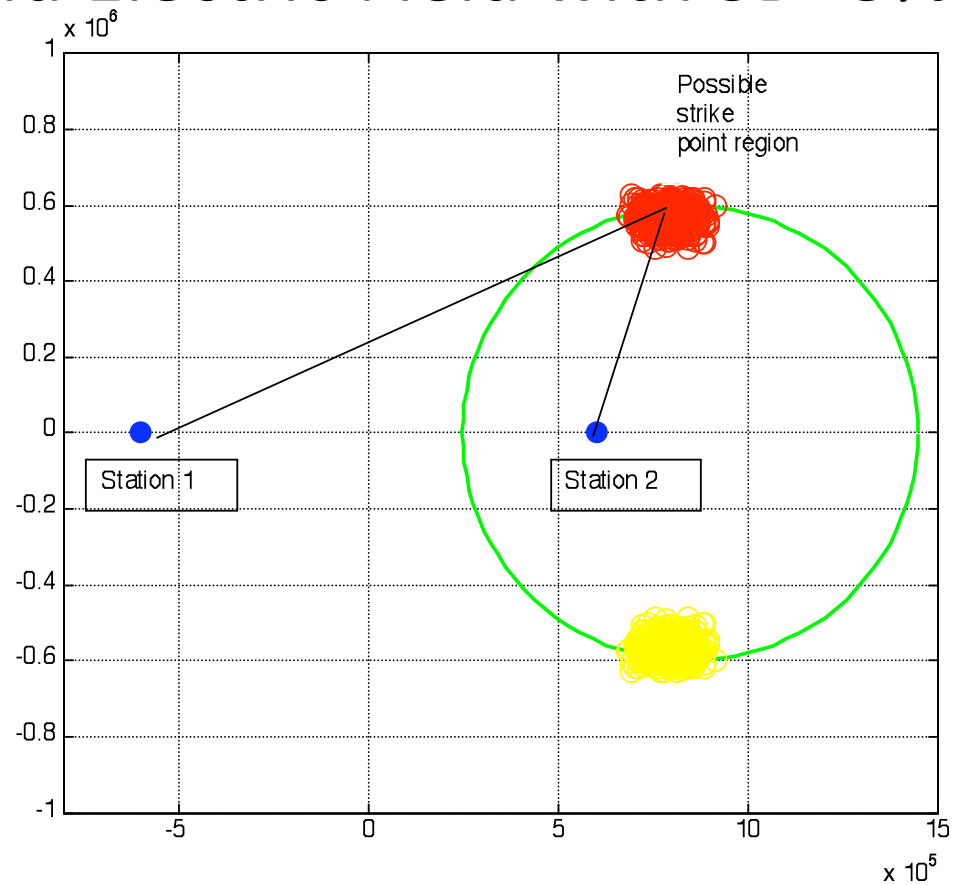
Monte Carlo Analysis for Time-Differences

- Time vectors with SD=5%
- Also position dependence



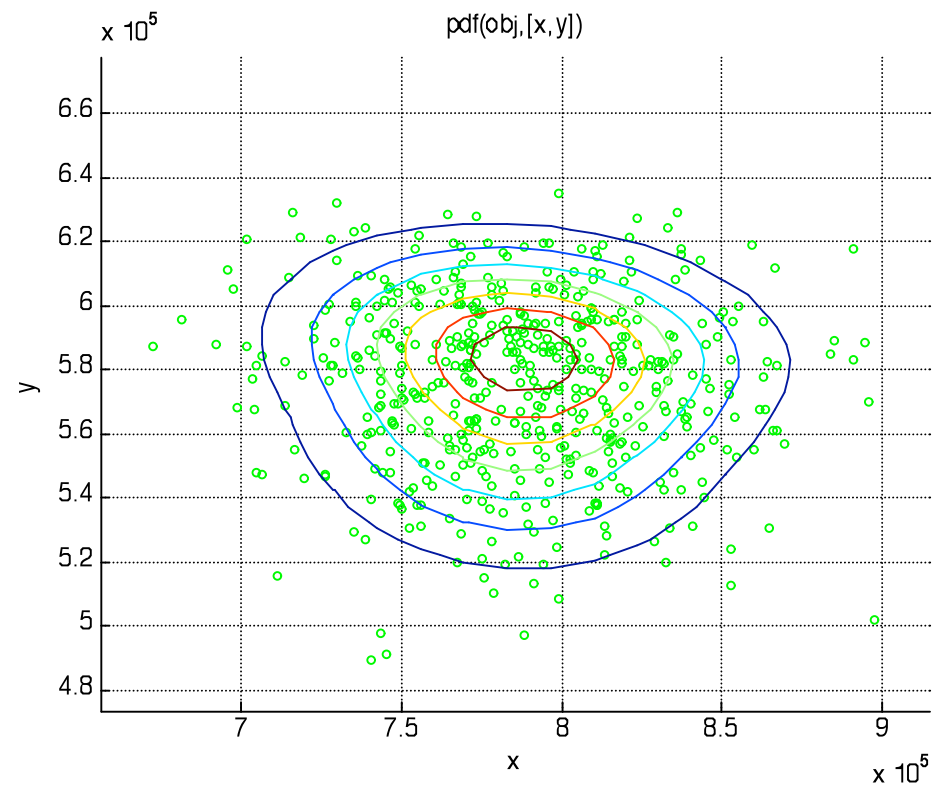
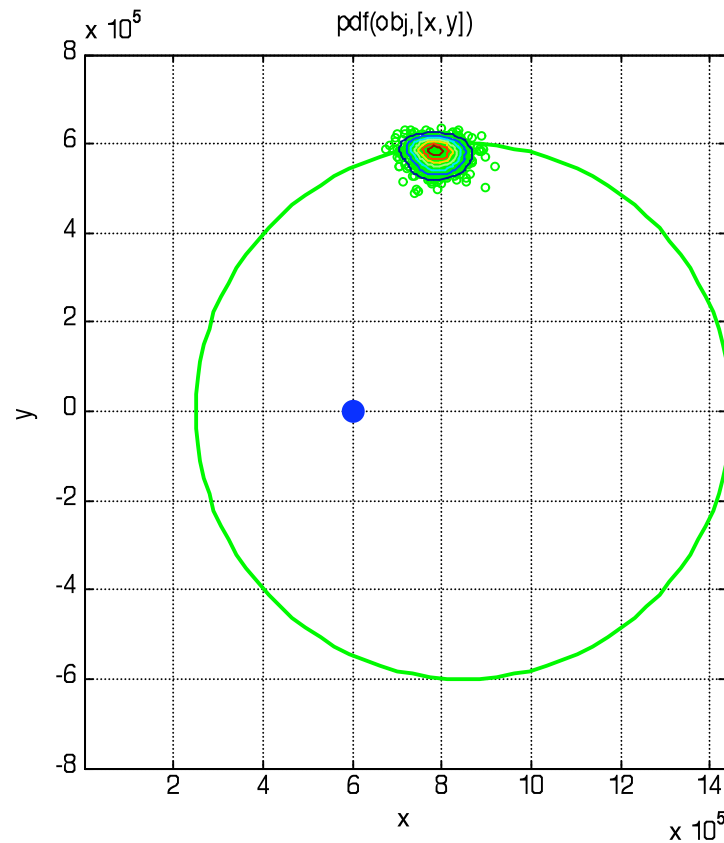
Combination

- Time difference and Electric Field with SD=5%
- Dependence on multiple variables



Error surfaces

- Error $x=6\%$, Error $y=4\%$ with $E_{\text{peak1}}/E_{\text{peak2}}=0.4$



Some advantages and disadvantages of the method

- Possible to use it as a 2-station method
- A disambiguation technique is required
- Relatively simple to implement
- Synchronization required for ToA part
- In $E = k/D^\eta$, get k out of the location process
- Exponent η might also be estimated under certain assumptions if a higher number of sensors is used