Motion of Electrons in an Electromagnetic Knot

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Electromagnetic knots:

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- A. F. Rañada and J. L. Trueba, *Phys. Lett. A* **202**, 337-342 (1995).
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- W. T. M. Irvine and D. Bouwmeester, *Nature Phys.* **4**, 716-720 (2008).

BL Model:

- A. F. Rañada and J. L. Trueba, *Nature* **383**, 32-33 (1996).
- A. F. Rañada, M. Soler and J. L. Trueba, *Phys. Rev. E* **62**, 7181-7190 (2000).

Magnetic lines

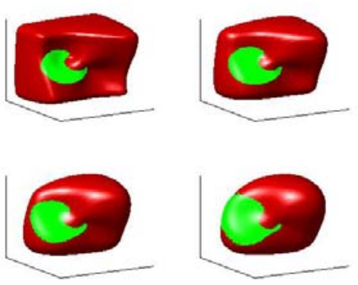
- Force lines of a magnetic field can be described in terms of a complex scalar field.

$$\mathrm{Re}\,\phi(\vec{r},t) = \mathrm{Re}\,\phi_0$$
$$\mathrm{Im}\,\phi(\vec{r},t) = \mathrm{Im}\,\phi_0$$

$$\vec{B} = f(\phi,\bar{\phi})\nabla\bar{\phi}\times\nabla\phi$$
$$\vec{E} = -f(\phi,\bar{\phi})\left(\frac{\partial\bar{\phi}}{\partial t}\nabla\phi - \frac{\partial\phi}{\partial t}\nabla\bar{\phi}\right)$$

- The electromagnetic field automatically satisfy two of Maxwell equations

$$\nabla\cdot\vec{B} = 0$$
$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\vec{E}$$



Magnetic knots

The Hopf index gives a tool to study the topology of the magnetic lines. So *f* is chosen as

$$f(\phi,\bar{\phi}) = \frac{-\sqrt{a}}{2\pi i\left(1+\phi\bar{\phi}\right)^2}$$

Here, *a* is a constant with the correct dimensions (action times velocity times magnetic permeability).

$$\vec{B} = \frac{\sqrt{a}}{2\pi i\left(1+\phi\bar{\phi}\right)^2}\nabla\phi\times\nabla\bar{\phi}$$

$$\vec{E} = \frac{\sqrt{a}}{2\pi i\left(1+\phi\bar{\phi}\right)^2}\left(\frac{\partial\bar{\phi}}{\partial t}\nabla\phi-\frac{\partial\phi}{\partial t}\nabla\bar{\phi}\right)$$

These are magnetic knots, in which the magnetic helicity is topologically quantized

$$h_m = \frac{1}{2\mu_0}\int\left(\vec{A}\cdot\vec{B}\right)d^3r = \frac{a}{2\mu_0}n \qquad \vec{B} = \nabla\times\vec{A}$$

Where *n* is the linking number of any two magnetic lines.

Magnetic knots II

Magnetic knots have to satisfy two of the Maxwell equations, leading to applications in plasma physics, ball lightning models, etc

$$\nabla \cdot \vec{E} = \frac{q}{\varepsilon_0}$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{j}$$

The magnetic helicity in these applications will be a constant of motion, and will have the topological meaning of linking number of magnetic lines.

The magnetic knots describe locally every situation in which the magnetic helicity is conserved.

Electromagnetic knots I

In vacuum, where no charges or currents are present, the electric field is divergenceless. We can then describe the electric lines as the level curves of another complex scalar field

$$\vec{B} = \frac{\sqrt{a}}{2\pi i\left(1+\phi\bar{\phi}\right)^2}\nabla\phi\times\nabla\bar{\phi} = \frac{\sqrt{a}/c}{2\pi i\left(1+\theta\bar{\theta}\right)^2}\left(\frac{\partial\bar{\theta}}{\partial t}\nabla\theta-\frac{\partial\theta}{\partial t}\nabla\bar{\theta}\right)$$

$$\vec{E} = \frac{\sqrt{a}}{2\pi i\left(1+\phi\bar{\phi}\right)^2}\left(\frac{\partial\bar{\phi}}{\partial t}\nabla\phi-\frac{\partial\phi}{\partial t}\nabla\bar{\phi}\right) = \frac{\sqrt{a}c}{2\pi i\left(1+\theta\bar{\theta}\right)^2}\nabla\bar{\theta}\times\nabla\theta \qquad (Def.1)$$

Magnetic and electric helicities are then conserved

$$\vec{B} = \nabla\times\vec{A} = \frac{1}{c^2}\frac{\partial\vec{C}}{\partial t}$$

$$\vec{E} = \nabla\times\vec{C} = -\frac{\partial\vec{A}}{\partial t}$$

$$h_m = \frac{1}{2\mu_0}\int\left(\vec{A}\cdot\vec{B}\right)d^3r = \frac{a}{2\mu_0}n(\phi)$$

$$h_e = \frac{\varepsilon_0}{2}\int\left(\vec{C}\cdot\vec{E}\right)d^3r = \frac{a}{2\mu_0}n(\theta)$$

Electromagnetic knots II

Electromagnetic fields in which the magnetic lines are the level curves of a complex scalar Φ, and the electric lines are the level curves of another complex scalar θ.

They can be written as in (*Def.* 1). If these equations hold, they satisfy automatically the Maxwell equations.

The electric and magnetic helicities of an electromagnetic knot are conserved quantities and they are proportional to the linking numbers of the magnetic and the electric lines, respectively.

Every solution of Maxwell equations in vacuum can be written locally as the sum of two electromagnetic knots.

The Hopf fibration I

$$\vec{B} = \frac{\sqrt{a}\lambda^2}{\pi\left(K^2+T^2\right)^3}\left(Q\overrightarrow{H_1}+P\overrightarrow{H_2}\right), \ \vec{E} = \frac{\sqrt{a}c\lambda^2}{\pi\left(K^2+T^2\right)^3}\left(Q\overrightarrow{H_2}-P\overrightarrow{H_1}\right)$$

$$P = T\left(T^2-3K^2\right), \ Q = K\left(K^2-3T^2\right), \ K = \frac{1}{2}\left(X^2+Y^2+Z^2-T^2+1\right)$$

$$\overrightarrow{H_1} = \left(Y+T-XZ, -X-(Y+T)Z, \frac{-1-Z^2+X^2+(Y+T)^2}{2}\right)$$

$$\overrightarrow{H_2} = \left(\frac{1+Z^2+X^2-(Y+T)^2}{2}, -Z+X(Y+T), XZ+(Y+T)\right)$$

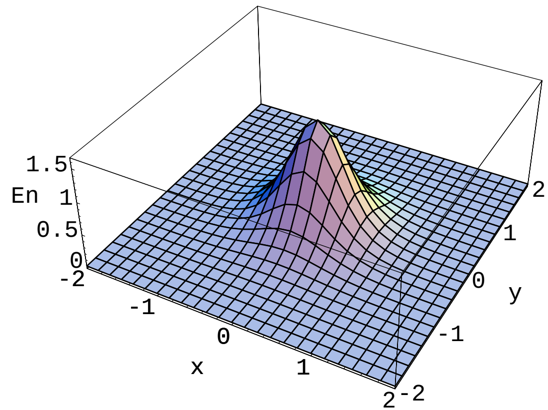
$$\left(X,Y,Z,T\right) = \lambda\left(x,y,z,ct\right)$$

The Hopf fibration II

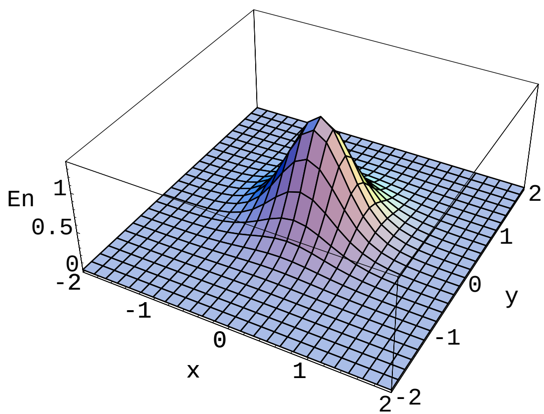
$$Energy = \frac{a\lambda}{\mu_0}$$

$$En.Dens.Unit = \frac{a\lambda^4}{\mu_0}$$

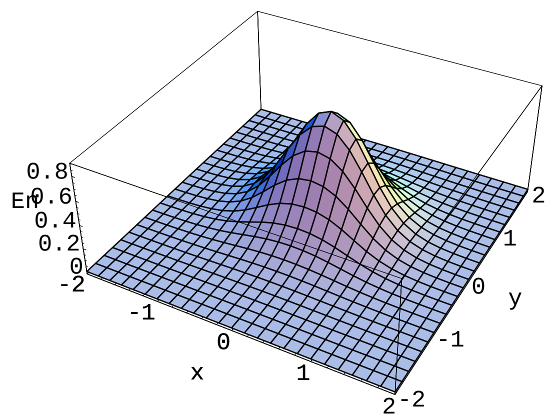
$$Maximum = (0, T(1+6T^2)/(2+6T^2), 0)$$



$T = 0$

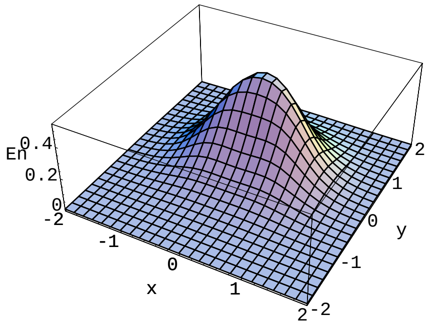


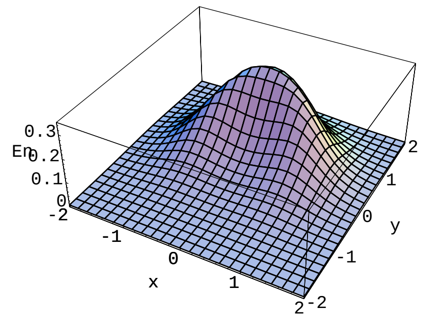
$T = 0.25$



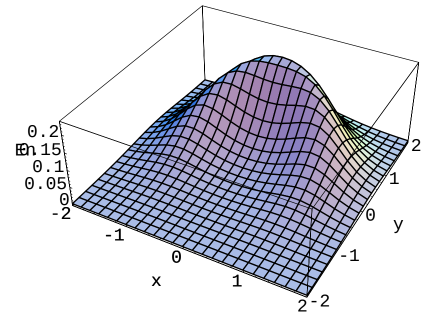
$T = 0.5$

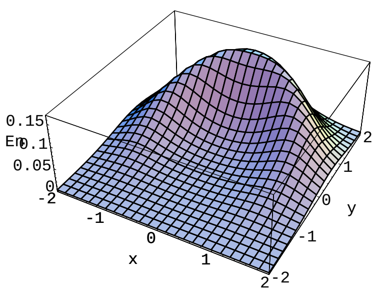
The Hopf fibration III

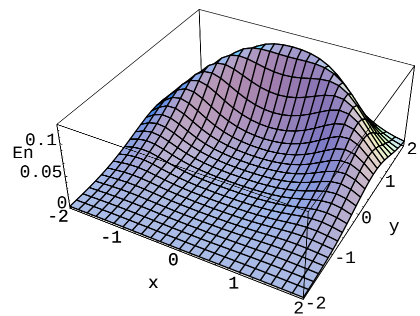


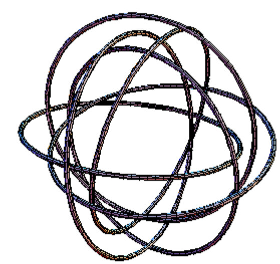


$T = 1$









Motion of electrons I

• Suppose that an electromagnetic knot has been created in certain region.
• Initially, the knot is centered at the origin, ant this center moves along the y-axis.
• Some free electrons with velocities much smaller than c are inside the ball initially.
• We study the motion of these electrons.

$$\frac{d\vec{v}}{dt} = -\frac{e}{m}\sqrt{1-\frac{v^2}{c^2}}\left(\vec{E}+\vec{v}\times\vec{B}-\frac{1}{c^2}\vec{v}\left(\vec{v}\cdot\vec{E}\right)\right)$$

$$v_0 \approx (10^{-2}-10^{-3})c$$

$$m = 9.1\times 10^{-31} kg$$

Motion of electrons II

- We use dimensionless quantities (upper-case symbols).
- We consider the electromagnetic knot given by the Hopf fibration.

$$\frac{d\vec{V}}{dT} = -\frac{e\lambda\sqrt{a}}{\pi mc}\sqrt{1-V^2}\left(\frac{Q\vec{H}_2 - P\vec{H}_1}{\left(K^2+T^2\right)^3} + \vec{V}\times\frac{Q\vec{H}_1 + P\vec{H}_2}{\left(K^2+T^2\right)^3} - \vec{V}\left(\vec{V}\cdot\frac{Q\vec{H}_2 - P\vec{H}_1}{\left(K^2+T^2\right)^3}\right)\right)$$

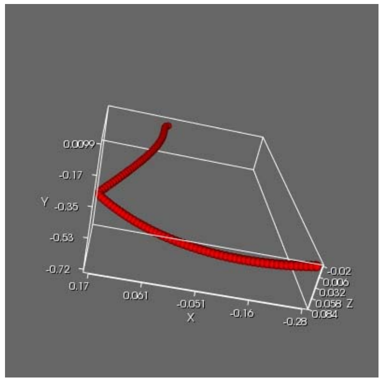
$$(X,Y,Z,T) = \lambda(x,y,z,ct)$$

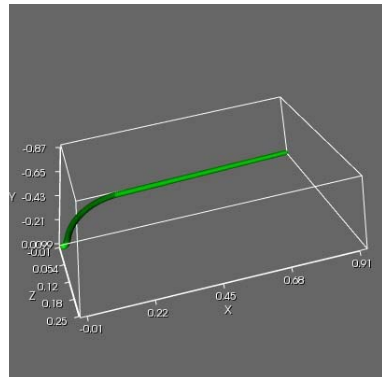
$$V = v/c$$

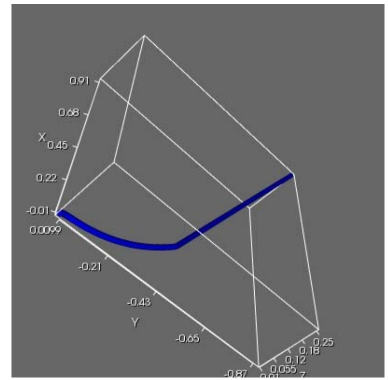
$$prefactor \approx 0.1\sqrt{Energy(J)/size(m)}$$

Motion of electrons III

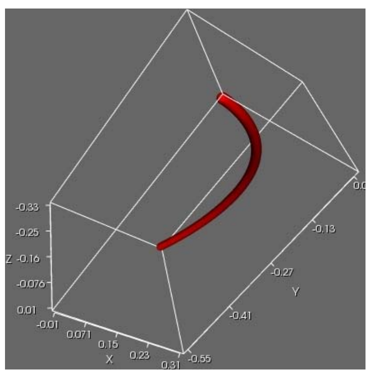
$prefactor = 100$

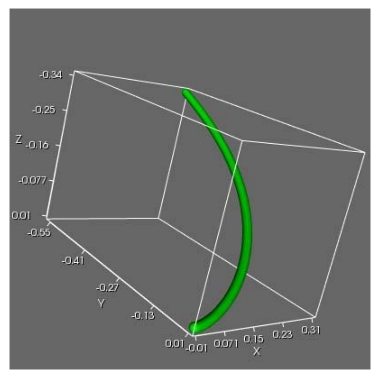


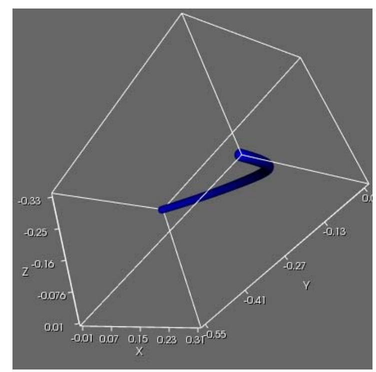




$prefactor = 1$







Conclusions

• Electromagnetic knots are solutions of Maxwell equations in vacuum such that their magnetic lines and their electric lines are the level curves of two complex scalar fields.

• Electromagnetic knots describe electromagnetic fields in which the energy is locallized in a given volume and with entanglement of field lines.

• With certain values of the electromagnetic energy and size of the knot, electrons that fall inside the knot can be acelerated to relativistic velocities.