Optical Signal Radiated from the Lightning Channel: Comparison of the Different Return Stroke Models

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Abstract:

- The intensity of the light pulse radiated from the lightning channel during the return stroke phase is analyzed.
- Accepting the exponential decay of the real lightning return stroke velocity, the time dependence of the apparent height and velocity with time is calculated by the causality consideration. The experimentally observed decrease of the apparent return stroke velocity with height for more than 30% is explained.
- Accepting the linear connection between the current in the lightning channel and the emitted optical signal during rise time of the current the total light intensity along the channel is calculated. The assumption that the heated gas is optically thin (no self-absorption of the emitted radiation) is accepted. The perfectly conducting ground is assumed with no current reflections from the striking point.
- Seven return stroke models are considered and compared: transmission line (TL), modified transmission line (MTL), Bruce-Golde (BG), traveling current source (TCS), Diendorfer-Uman (DU), modified DU (MDU) and the generalized traveling current source model GTCS.

The Apparent Height and the Apparent Return Stroke Velocity

- For the computation of the return stroke radiation pattern it is necessary to know the so-called apparent height and the apparent return stroke wave-front velocity.
- They represent the height and the velocity of the return stroke wave-front seen from the observer's point in some instant of time during the channel discharge. The existence of straight vertical channel without tortuosity is assumed.

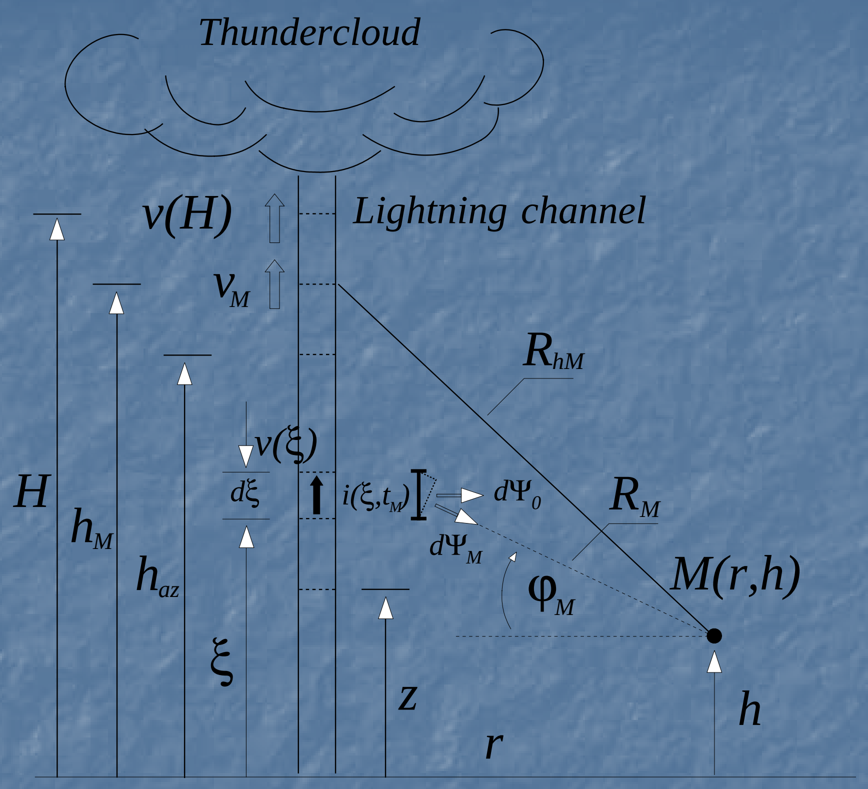


Fig.1 The geometry of the problem

Theory:

$$\tau_u(z) = \int_0^z d\xi / v(\xi), \quad \tau_{rad}(z) = R_{\mathrm{M}} / c$$

$$\tau = \tau_u + \tau_{rad}\ , \quad \tau = t\ , \quad z = h_{\mathrm{M}}\ , \quad R_{\mathrm{M}} = R_{\mathrm{hM}}$$

$$\int_0^{h_{\mathrm{M}}} d\xi / v(\xi) = t - R_{\mathrm{hM}}, \qquad (1)$$

$$R_{\mathrm{hM}} = \sqrt{r^2 + (h_{\mathrm{M}} - h)^2}$$

Total light intensity emitted from the channel

$$\Psi_{\mathrm{M}}(t_{\mathrm{M}}) = \int_0^{h_{\mathrm{M}}} (r / R_{\mathrm{M}}^3)\, i_{\mathrm{M}}(\xi, t_{\mathrm{M}})\, d\xi\ , \qquad (2)$$

$$i_{\mathrm{M}}(\xi, t_{\mathrm{M}}) = i(\xi,\ t_{\mathrm{M}} + t_{\min} - R_{\mathrm{M}} / c)$$

Example of the Profile of the Return Stroke Velocity

$$v(z) = v_0 \exp(-z/\Lambda) \quad (3)$$

Solving Eq.(1) using Eq.(2) one obtains

$$h_{\mathrm{M}} = \Lambda \ln[1 + v_0(t - R_{\mathrm{hM}}/c)/\Lambda]$$

The average return stroke velocity along the channel

$$v_{av}(z) = z \Big/ \int_0^z d\xi / v(\xi),$$

The average apparent velocity seen from the observer's point M

$$v_{av\mathrm{M}} = h_{\mathrm{M}} / t_{\mathrm{M}}, \qquad t_{\mathrm{M}} = t - \sqrt{r^2 + h^2}/c$$

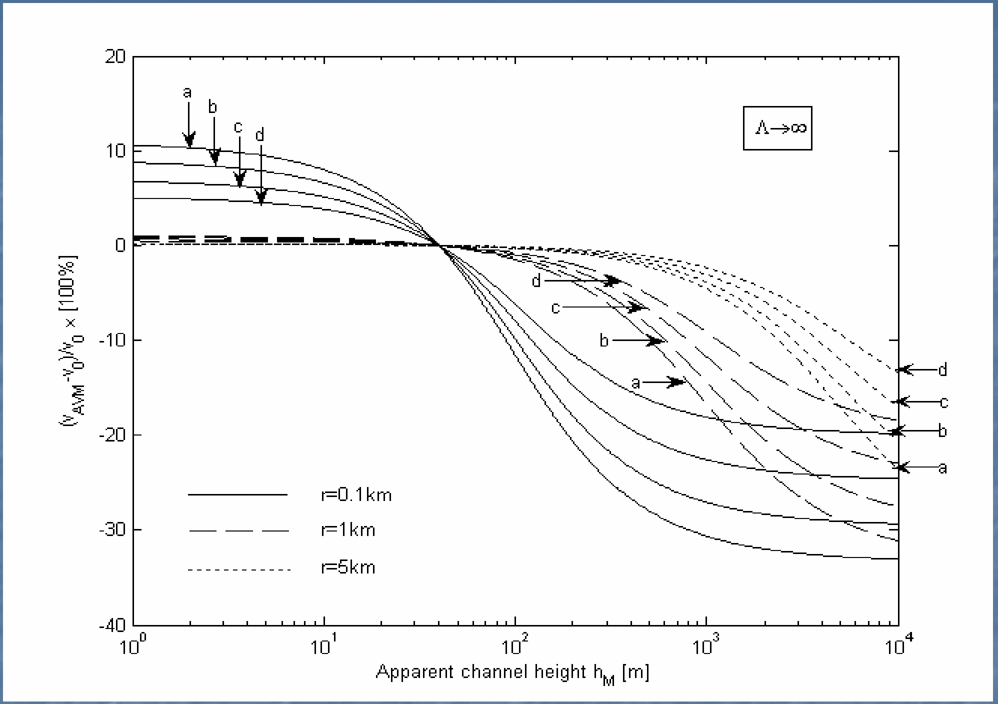


Fig.2 The deviation of the average apparent return stroke velocity as a function of the apparent return stroke height at h=20m.
a) v_0=0.5c, b) v_0=0.42c, c) v_0=0.33c, d) v_0=0.25c. The real return stroke velocity is constant.

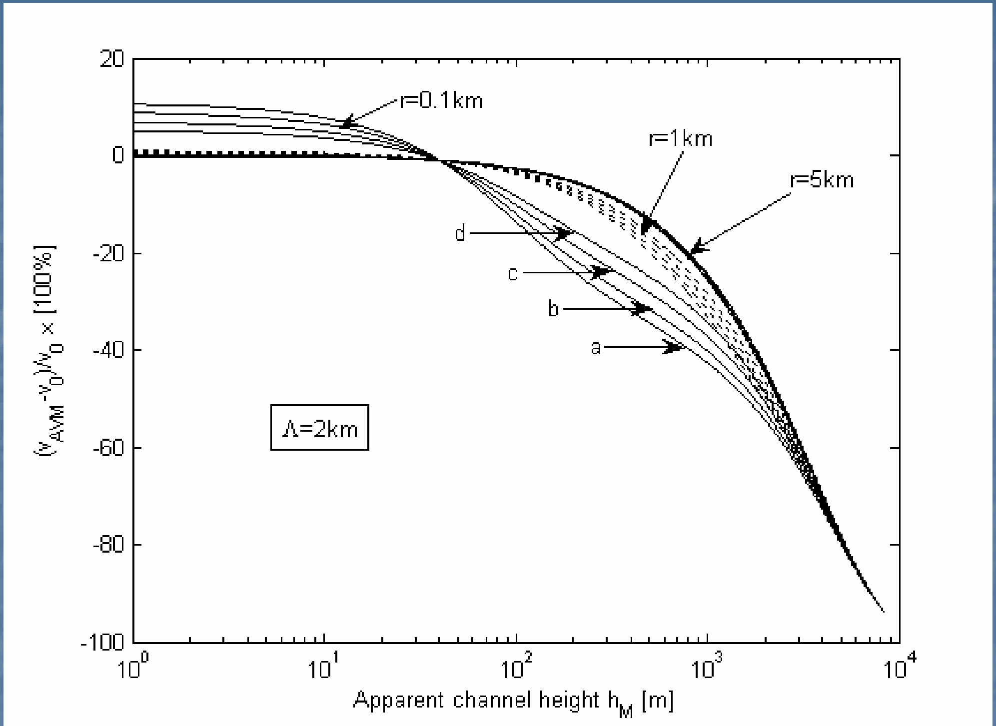


Fig.3 The deviation of the average apparent return stroke velocity as a function of the apparent return stroke height at h=20m. a) v_0=0.5c, b) v_0=0.42c, c) v_0=0.33c, d) v_0=0.25c. The real return stroke velocity decreases exponentially.

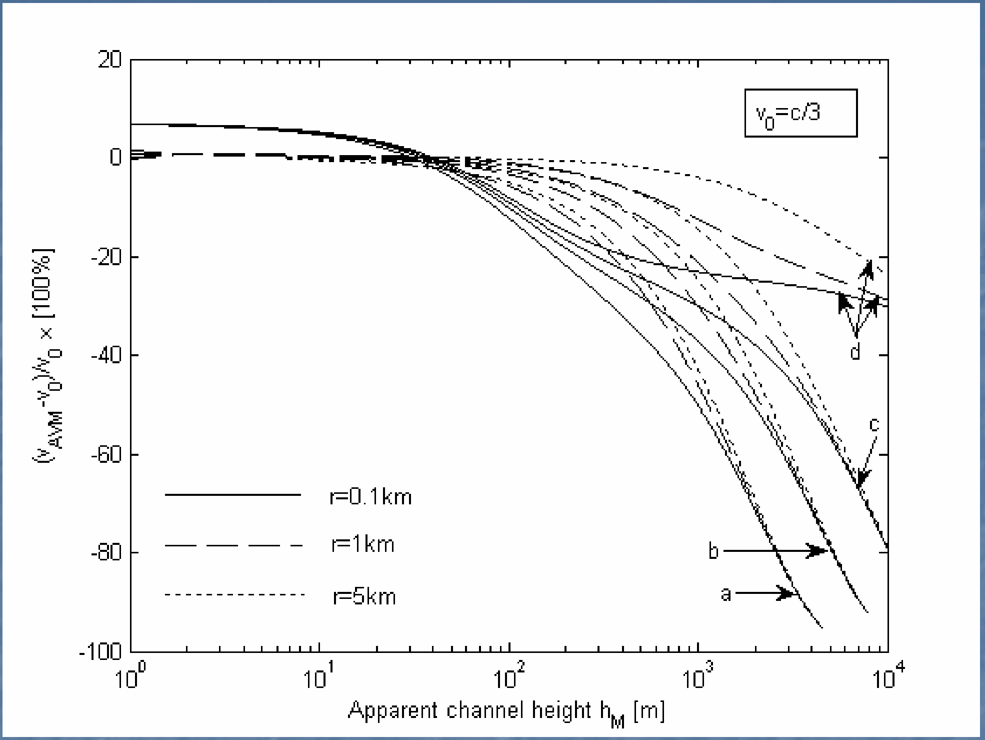


Fig.4 The deviation of the average apparent return stroke velocity as a function of the apparent return stroke height at h=20m. The real return stroke velocity decreases exponentially. a) Λ=1 km, b) Λ =2 km, c) Λ =4km, d) Λ → ∞.

Channel-base current

$$i_0(t) = (I/\eta)t^n / (t^n + \tau_1^n)\exp(-t/\tau_2) \quad (3)$$

Table 1. Typical lightning current quantities of the negative return stroke measured at the top of the tower (according to Berger et al, 1975, Anderson & Erikson, 1980)

i_{0max} (kA)	$(di/dt)_{max}$ (kA/μs)	τ_p (μs)	Q_0 (C)
29.7	23	4.2	2.34

Table 2. The channel-base current parameters according to Eq.(3) and the measurements given in Table 1. (Cvetic et al, 2000)

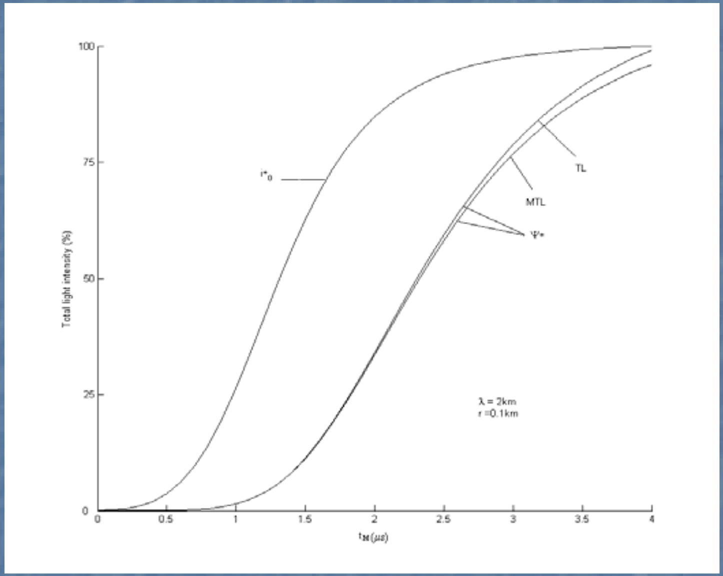
I (kA)	η	n	τ_1 (μs)	τ_2 (μs)	τ_d (μs) (DU)	λ(m) MTL
29.7	0.946	4.83	1.66	75.7	0.6	2000

TL model (Uman & McLain, 1969)

$$i(z,t) = i_0(t - z / v_{av})\, u(t - z / v_{av}),\quad u - \text{Heaviside (unit) function}$$

MTL model (Nucci et al, 1988)

$$i(z,t) = i_0(t - z / v_{av}) \exp(-z / \lambda)\, u(t - z / v_{av})$$



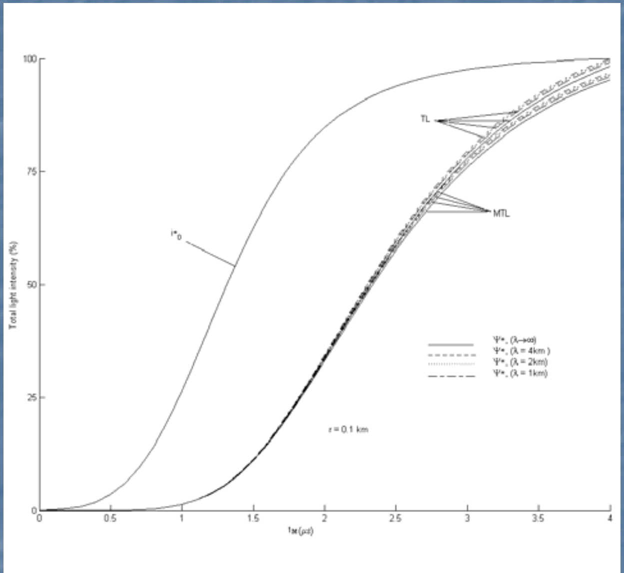


Fig.5 The total light intensity emitted during the rise time of the channel-base current for close distance r=0.1km, according to the TL and MTL models. The real return stroke velocity decreases exponentially.

BG model (Bruce & Golde, 1941)

$$i(z,t) = i_0(t)\,u(t - z/v_{av})$$

TCS model (Heidler, 1985)

$$i(z,t) = i_0(t + z/c)\,u(t - z/v_{av})$$

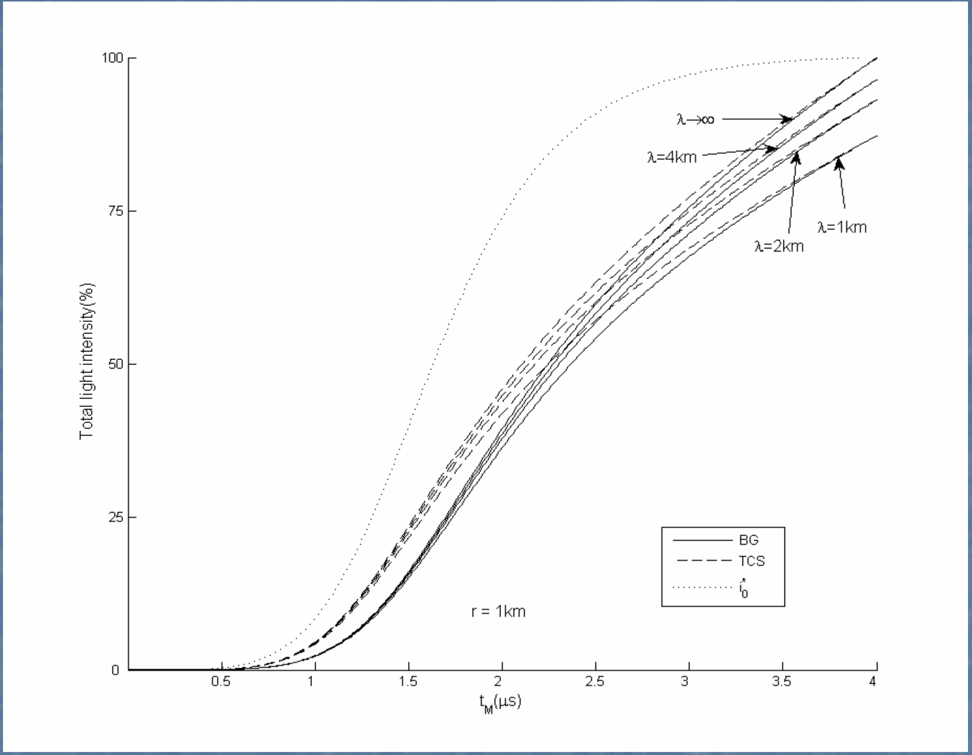


Fig.6 The total light intensity emitted during the rise time of the channel-base current for medium distance r=1km, according to the BG and TCS models. The real return stroke velocity decreases exponentially.

DU model (Diendorfer & Uman, 1990)

$$i(z,t) = \{i_0(t+z/c) - i_0(z/v)\exp[-(t-z/v)/\tau_d]\}\,u(t-z/v)$$

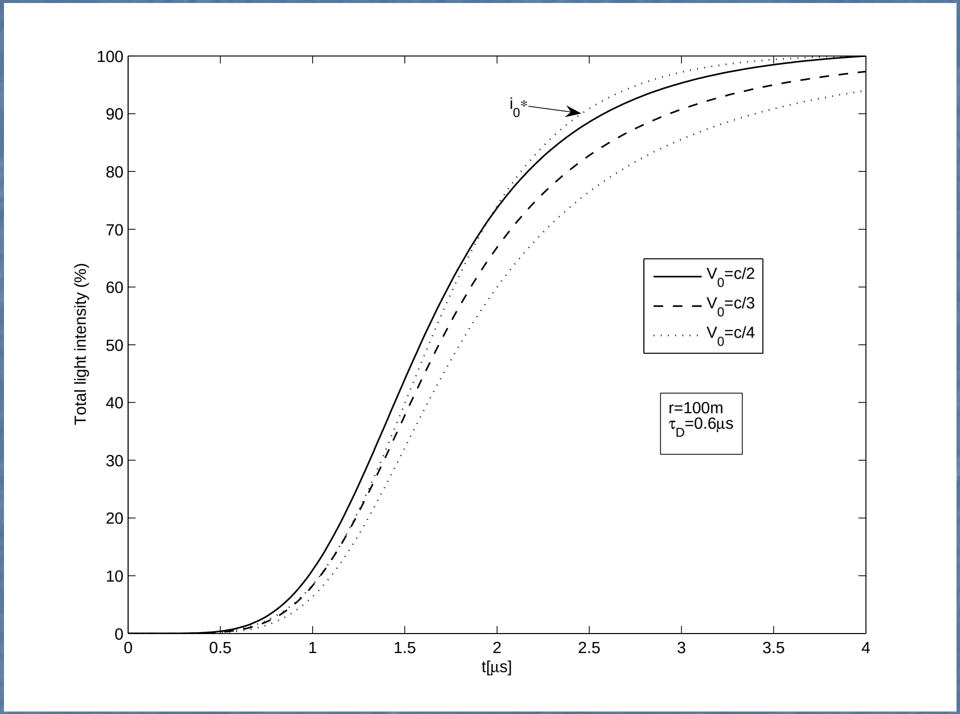


Fig.7 The total light intensity emitted during the rise time of the channel-base current for close distance r=0.1km, according to the DU model. The real return stroke velocity is constant and chosen as a parameter.

MDU model (Thottappillil et al, 1991)

$$i(z,t) = \{i_0(t + z/c) - i_0(z/v_{av}^*)\exp[-(t - z/v_{av})/\tau_d]\}\, u(t - z/v_{av})$$

$$v_{av}^* = v_{av}c/(v_{av} + c)$$

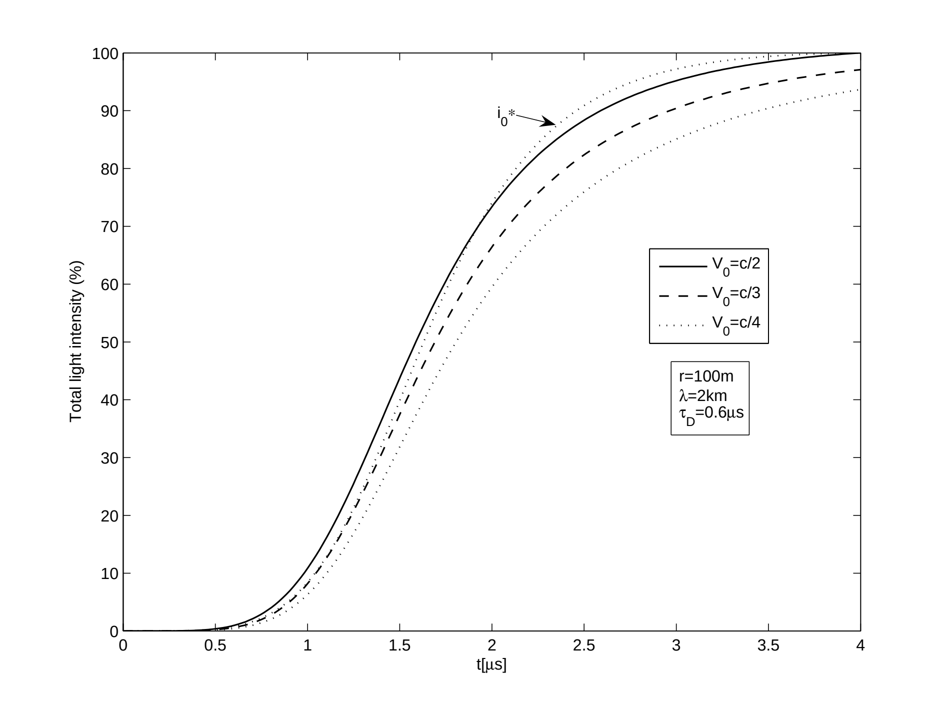


Fig.8 The total light intensity emitted during the rise time of the channel-base current for close distance r=0.1km, according to the MDU models. The real return stroke velocity decreases exponentially.

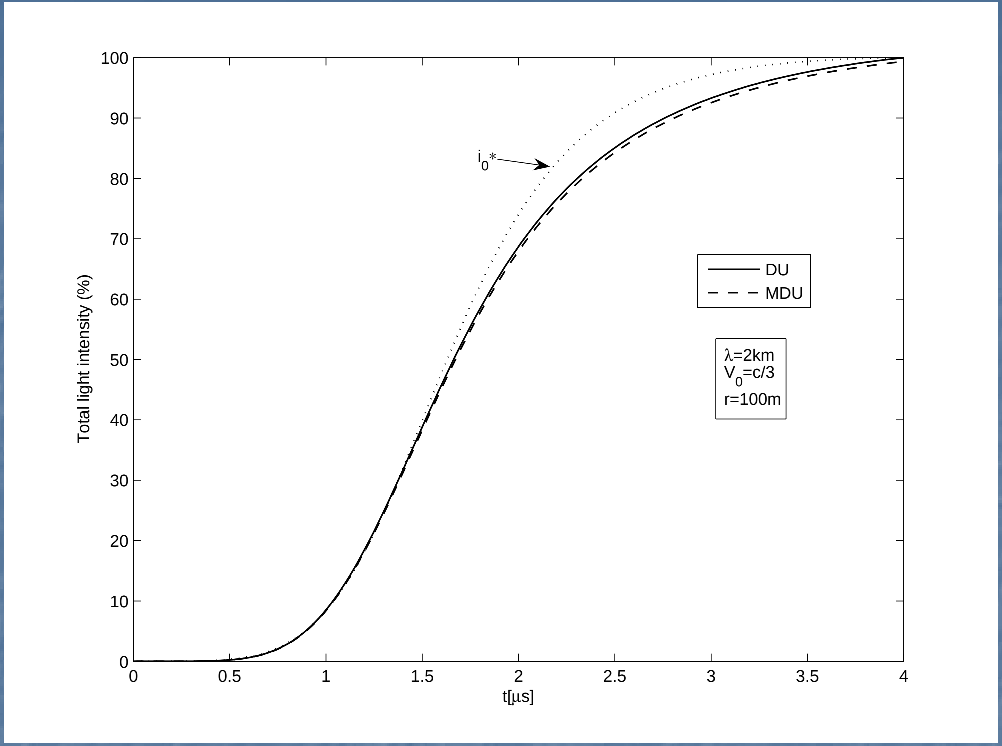


Fig.9 The comparison of the total light intensity emitted during the rise time of the channel-base current for close distance r=0.1km, according to the DU and MDU models.

GTCS model (Cvetic & Stanic, 1995)

- The current at the channel base and the initial charge distribution along the channel are considered as known. The channel discharge function *f* is calculated.

$$q'(z,t) = q_0'(z,t)\, f(z, t - z/v), \qquad t \geq z/v$$

$$f(z, t - z/v) = 1 + \int_0^{t - z/v} f_1(z,\xi)\, d\xi$$

$$f_1 = F^{-1}\left[I_0(s) / Q_0'(s/v^*) \right], \quad v^* = vc/(v + c)$$

$$i(z,t) = \int_z^{h_{az}} q_0'(\xi) \frac{\partial}{\partial t} f(z, t - \xi / v^* + z/c)\, d\xi \,,$$

$$h_{az} = v^*(t + z/c)$$

$$(1)\quad f(z,u=0)=1,\qquad u=t-z/v$$

$$(2)\quad f(z,u\to\infty)=0$$

$$(3)\quad f(z,u\geq 0)\geq 0$$

$$(4)\quad \partial f(z,u)/\partial u\,|_{u\geq 0}<0$$

- Assumed analytical form of the initial charge distribution function

$$q_0^{'}(z)=Q_{01}^{'}\left[g(z)+\lambda_d(z)\frac{dg}{dz}\right],\quad g(z)=\frac{z^m}{z^m+\lambda_1^m}\exp(-z/\lambda_2)\qquad(4)$$

Table 3. The values of the parameters of the initial channel charge distribution according to Eq.(4)

Curve	Q'_{01} (mC/m)	λ_1 (m)	λ_2 (m)	λ_d (m)	m
a)	8.2	5	300	0	1
b)	9.1	2	257	5	3

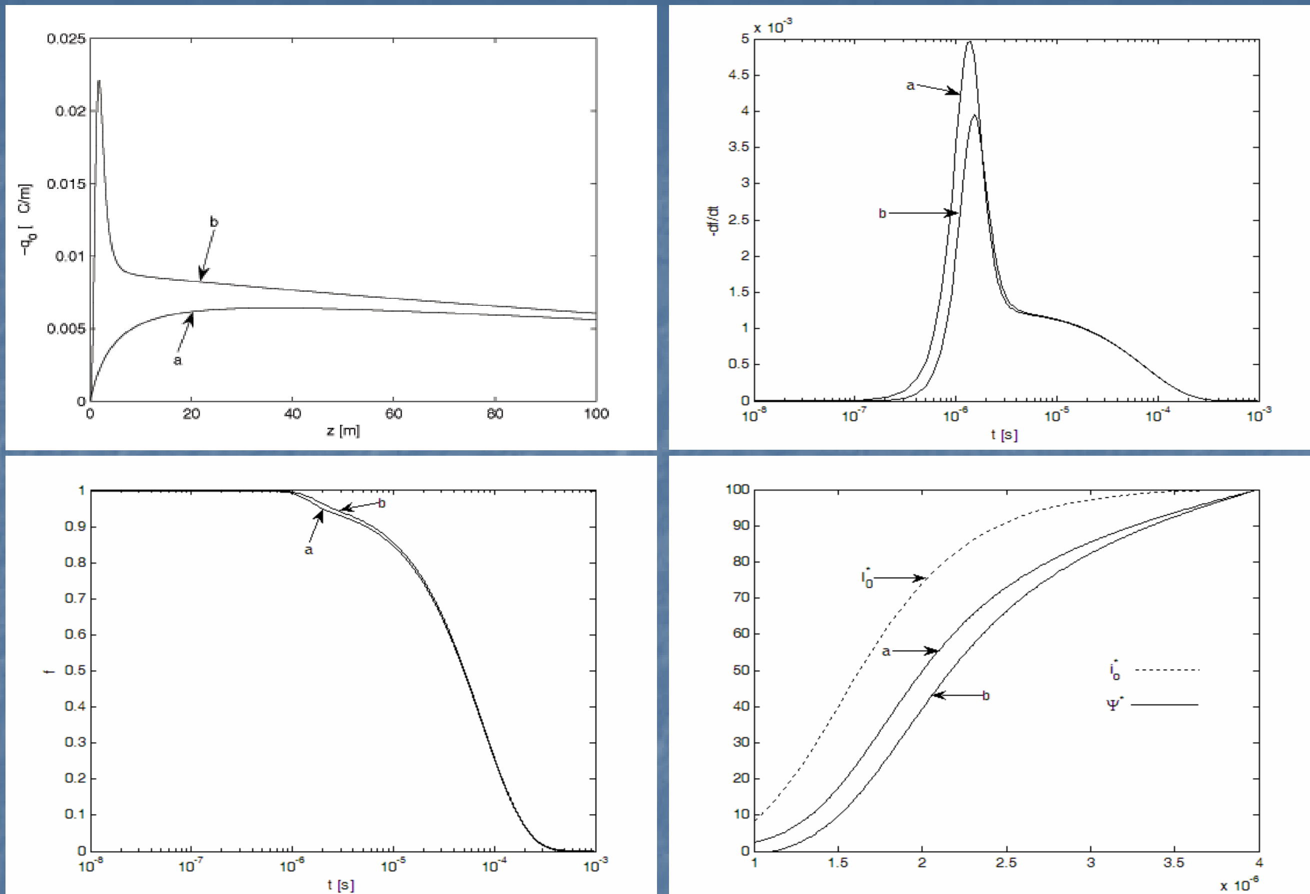


Fig.9 The total light intensity emitted during the rise time of the channel-base current for close distance r=0.1km, according to the GTCS model. The real return stroke velocity has the constant value, v_0=0.33c.