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Lightning Electromagnetic Field Computation

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Outline

Description of the problem

Dipole and monopole methods for field calculations

Non-uniqueness of field components

Special case of speed of light for return stroke speed and traveling pulse

Commonly used return stroke models

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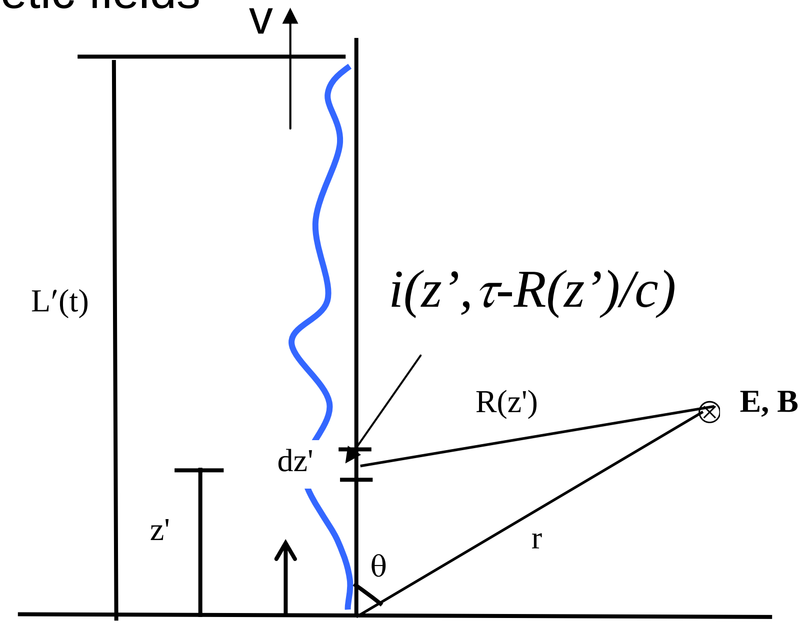
Description of the problem

Lightning return stroke speed 1-2x10^8 m/s

Current rise time < 10^{-6} s

Distributed source fast changing in both space and time

Methods of finding exact expressions for remote electric and magnetic fields



L’(t) < v.t

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Proper treatment of retardation effects

Limits of integration

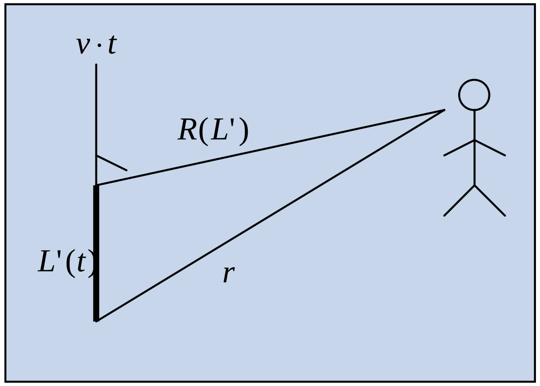
$$t = \frac{L'(t)}{v} + \frac{R(L')}{c} \qquad R(L') = \sqrt{r^2 + L'^2(t) - 2L'(t)r\cos\theta}$$

$$L'(t) = \frac{r}{1-(v^2/c^2)}\left(-\frac{v^2}{c^2}\cos\theta + \frac{vt}{r} - \frac{v}{c}\sqrt{\left(1-\frac{v^2}{c^2}\right) + \frac{v^2t^2}{r^2} + \frac{v^2}{c^2}\cos^2\theta - \frac{2vt}{r}\cos\theta}\right)$$

$$\frac{dL'}{dt} = \frac{v}{1-\frac{v}{c}\cos\theta(L')}$$, apparent speed of return stroke wavefront

Very far away $L'(t) = \dfrac{v}{1-\frac{v}{c}\cos\theta}\cdot(t - r/c)$

F-factor, $F = \left[1-\frac{v}{c}\cos\theta\right]^{-1}$



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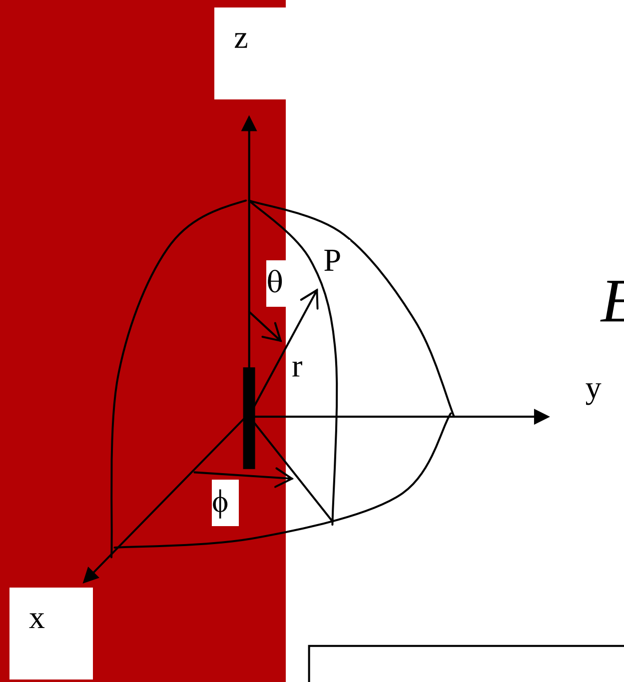
Electric fields from a dipole



Static

$$E_r = \frac{Z_0}{2\pi} m_E \cos\theta \left(\frac{1}{r^2} + \frac{1}{j\beta r^3} \right) e^{-j(\beta r - \omega t)}$$

$$E_\theta = \frac{-jZ_0\beta}{4\pi} m_E \sin\theta \left(\frac{1}{r} + \frac{1}{j\beta r^2} - \frac{1}{\beta^2 r^3} \right) e^{-j(\beta r - \omega t)}$$

$$H_\phi = \frac{j\beta}{4\pi} m_E \sin\theta \left(\frac{1}{r} + \frac{1}{j\beta r^2} \right) e^{-j(\beta r - \omega t)}$$

Are the field components unique?

Radiation

Induction or intermediate

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Electric fields from line sources

(Line source – several dipoles connected end to end)

Is it possible to define static, induction and radiation components uniquely?

How do we define static field? 1/distance^3 ?

How do we define radiation field? 1/distance ?

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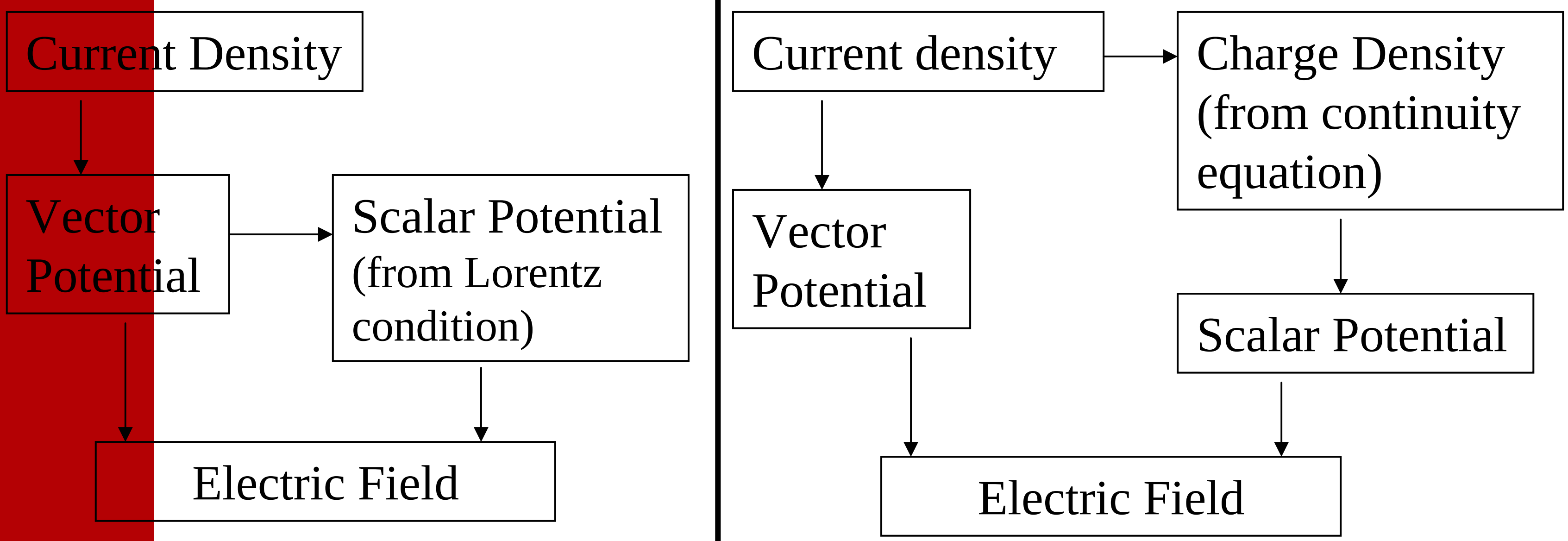
Two methods for finding electric fields

Monopole method



(Explicit use of continuity equation)

(Jefimenko)

Dipole method

(Explicit use of Lorentz condition)

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Dipole and monopole methods for field calculations - 1

Dipole method

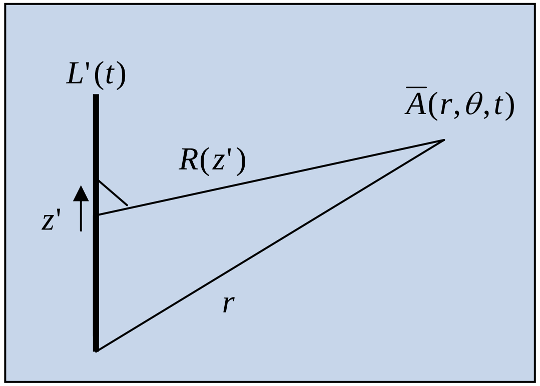
(The Lorentz condition approach)

$$\overline{A}(r,\theta,\tau) = \frac{1}{4\pi\varepsilon_0 c^2}\int_0^{L'(\tau)} \frac{i\,(z',\tau - R(z')/c)}{R(z')}\hat{z}dz'$$

$$\nabla \bullet \overline{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$$

$$\phi(r,\theta,t) = -c^2 \int_{r/c}^{t} \nabla \bullet \overline{A} d\tau$$

(Lorentz condition)



$$\overline{E} = -\nabla \phi - \frac{\partial \overline{A}}{\partial t}$$

$$\overline{B} = \nabla \times \overline{A}$$

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Dipole and monopole methods for field calculations - 2

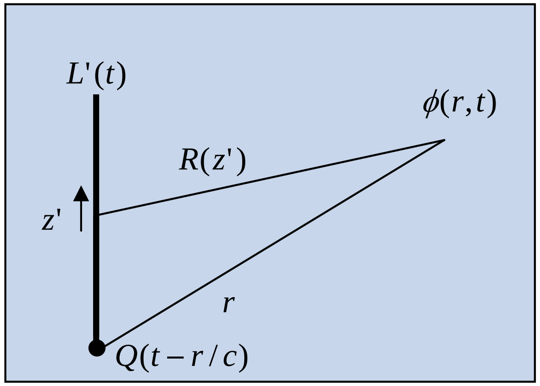
The monopole method

(The continuity equation approach)

$$\bar{A}(r,\theta,\tau) = \frac{1}{4\pi\varepsilon_0 c^2}\int_0^{L'(\tau)} \frac{i\,(z',\tau - R(z')/c)}{R(z')}\hat{z}dz'$$

$$\frac{\partial\rho^*(z',t-R(z')/c)}{\partial t} = -\left.\frac{\partial i(z',t-R(z')/c)}{\partial z'}\right|_{t-R(z')/c=const.}$$

$$\phi(r,t) = \frac{1}{4\pi\varepsilon_0}\frac{Q(t-r/c)}{r} + \frac{1}{4\pi\varepsilon_0}\int_0^{L'(t)} \frac{1}{R(z')}\rho^*(z',t-R(z')/c)dz'$$



$$\bar{E} = -\nabla\phi - \frac{\partial\bar{A}}{\partial t}$$

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Expressions for scalar potential

$$\phi(r,t) = -\frac{1}{4\pi\varepsilon_0}\int_0^{L'(t)} \left[\frac{z'}{R^3(z')} \int_{z'/v+R(z')/c}^{t} i\left(z',\tau-\frac{R(z')}{c}\right)d\tau + \frac{z'}{cR^2(z')}\, i\left(z',t-\frac{R(z')}{c}\right)\right] dz'.$$

Dipole method

$$\phi(r,t) = \frac{1}{4\pi\varepsilon_0}\frac{Q(t-r/c)}{r} + \frac{1}{4\pi\varepsilon_0}\int_0^{L'(t)} \frac{1}{R(z')}\rho^*\left(z',t-R(z')/c\right)dz',$$

Monopole method

$$Q(t-r/c) = -\int_{r/c}^{t} i(0,\tau-r/c)\,d\tau$$

cost

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General and exact E field expression – Dipole method

$$\bar{E}(r,\theta,t) = -\frac{1}{4\pi\varepsilon_0}\hat{r}\int_0^{L'(t)}\frac{\cos\theta - 3\cos\alpha(z')\cos\beta(z')}{R^3(z')}\int_{t_b}^{t} i(z',\tau-\frac{R(z')}{c})d\tau\, dz' \qquad (a)$$

$$-\frac{1}{4\pi\varepsilon_0}\hat{r}\int_0^{L'(t)}\frac{\cos\theta - 3\cos\alpha(z')\cos\beta(z')}{cR^2(z')}i(z',t-\frac{R(z')}{c})dz' \qquad (b)$$

$$-\frac{1}{4\pi\varepsilon_0}\hat{r}\int_0^{L'(t)}\frac{\cos\theta - \cos\alpha(z')\cos\beta(z')}{c^2R(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}dz' \qquad (c)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{\theta}\int_0^{L'(t)}\frac{\sin\theta + 3\cos\alpha(z')\sin\beta(z')}{R^3(z')}\int_{t_b}^{t} i(z',\tau-\frac{R(z')}{c})d\tau\, dz' \qquad (d)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{\theta}\int_0^{L'(t)}\frac{\sin\theta + 3\cos\alpha(z')\sin\beta(z')}{cR^2(z')}i(z',t-\frac{R(z')}{c})dz' \qquad (e)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{\theta}\int_0^{L'(t)}\frac{\sin\theta + \cos\alpha(z')\sin\beta(z')}{c^2R(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}dz' \qquad (f)$$

$$-\frac{1}{4\pi\varepsilon_0}\hat{r}\frac{\cos\theta - \cos\alpha(L')\cos\beta(L')}{c^2R(L')}i(L',t-\frac{R(L')}{c})\frac{dL'}{dt} \qquad (g)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{\theta}\frac{\sin\theta + \cos\alpha(L')\sin\beta(L')}{c^2R(L')}i(L',t-\frac{R(L')}{c})\frac{dL'}{dt} \qquad (h)$$

General and exact E field expression – Monopole method

$$\bar{E}(r,\theta,t) = +\frac{1}{4\pi\varepsilon_0}\hat{r}\int_0^{L'(t)}\frac{\cos\beta(z')}{R^2(z')}\rho^*(z',t-R(z')/c)\,dz' \qquad (a)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{r}\int_0^{L'(t)}\frac{\cos\beta(z')}{cR(z')}\frac{\partial\rho^*(z',t-R(z')/c)}{\partial t}dz' \qquad (b)$$

$$-\frac{1}{4\pi\varepsilon_0}\hat{r}\int_0^{L'(t)}\frac{\cos\theta}{c^2R(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}dz' \qquad (c)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{\theta}\int_0^{L'(t)}\frac{\sin\beta(z')}{R^2(z')}\rho^*(z',t-R(z')/c)\,dz' \qquad (d)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{\theta}\int_0^{L'(t)}\frac{\sin\beta(z')}{cR(z')}\frac{\partial\rho^*(z',t-R(z')/c)}{\partial t}dz' \qquad (e)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{\theta}\int_0^{L'(t)}\frac{\sin\theta}{c^2R(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}dz' \qquad (f)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{r}\frac{\cos\beta(L')}{cR(L')}\rho^*(L',t-R(L')/c)\frac{dL'(t)}{dt} - \frac{1}{4\pi\varepsilon_0}\hat{r}\frac{\cos\theta}{c^2R(L')}i(L',t-R(L')/c)\frac{dL'(t)}{dt} \qquad (g),(h)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{\theta}\frac{\sin\beta(L')}{cR(L')}\rho^*(L',t-R(L')/c)\frac{dL'(t)}{dt} + \frac{1}{4\pi\varepsilon_0}\hat{\theta}\frac{\sin\theta}{c^2R(L')}i(L',t-R(L')/c)\frac{dL'(t)}{dt} \qquad (i),(j)$$

$$+\frac{1}{4\pi\varepsilon_0}\hat{r}\frac{1}{r^2}Q(t-r/c) + \frac{1}{4\pi\varepsilon_0}\hat{r}\frac{1}{rc}\frac{dQ(t-r/c)}{dt} \qquad (k),(l)$$

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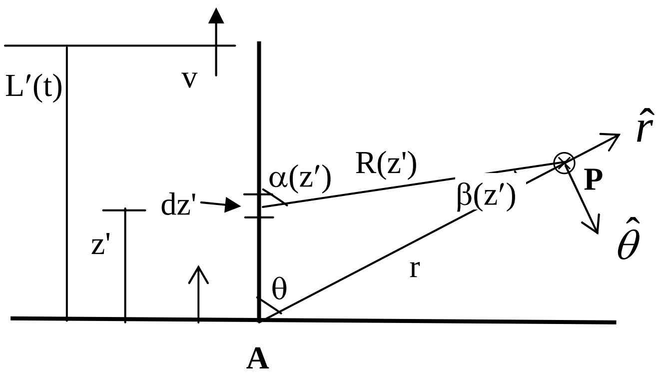
General and exact expression for magnetic field

$$B(r,t) = \frac{1}{4\pi\varepsilon_0 c^2}\hat{\phi}\int_0^{L'(t)} \left(\frac{\sin\alpha(z')}{R^2(z')} i(z',t-R(z')/c) + \frac{\sin\alpha(z')}{cR(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}\right)dz'$$
$$+ \frac{1}{4\pi\varepsilon_0 c^2}\hat{\phi}\frac{\sin\alpha(L')}{cR(L')} i(L',t-R(L')/c)\frac{dL'}{dt}$$



Sample calculation using the two methods

(lightning return stroke field at ground)

Dipole method

$$i(z',t) = i(0, t - z'/v)$$

$$E_V(r,t) = \frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{2-3\sin^2\alpha(z')}{R^3(z')}\int_{t_b}^{t} i\,(z',\tau - R(z')/c)d\tau\,dz'$$

$$+\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{2-3\sin^2\alpha(z')}{cR^2(z')} i(z',t-R(z')/c)\,dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{\sin^2\alpha(z')}{c^2R(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}dz'$$

Monopole method

$$E_V(r,t) = -\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{z'}{R^3(z')}\rho^*(z',t-R(z')/c)\,dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{z'}{cR^2(z')}\frac{\partial \rho^*(z',t-R(z')/c)}{\partial t}dz'$$

$$-\frac{1}{2\pi\varepsilon_0}\hat{z}\int_0^{L'(t)} \frac{1}{c^2R(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}dz'$$

Static $\frac{1}{R^3}$

Induction $\frac{1}{cR^2}$

Radiation $\frac{1}{c^2R}$

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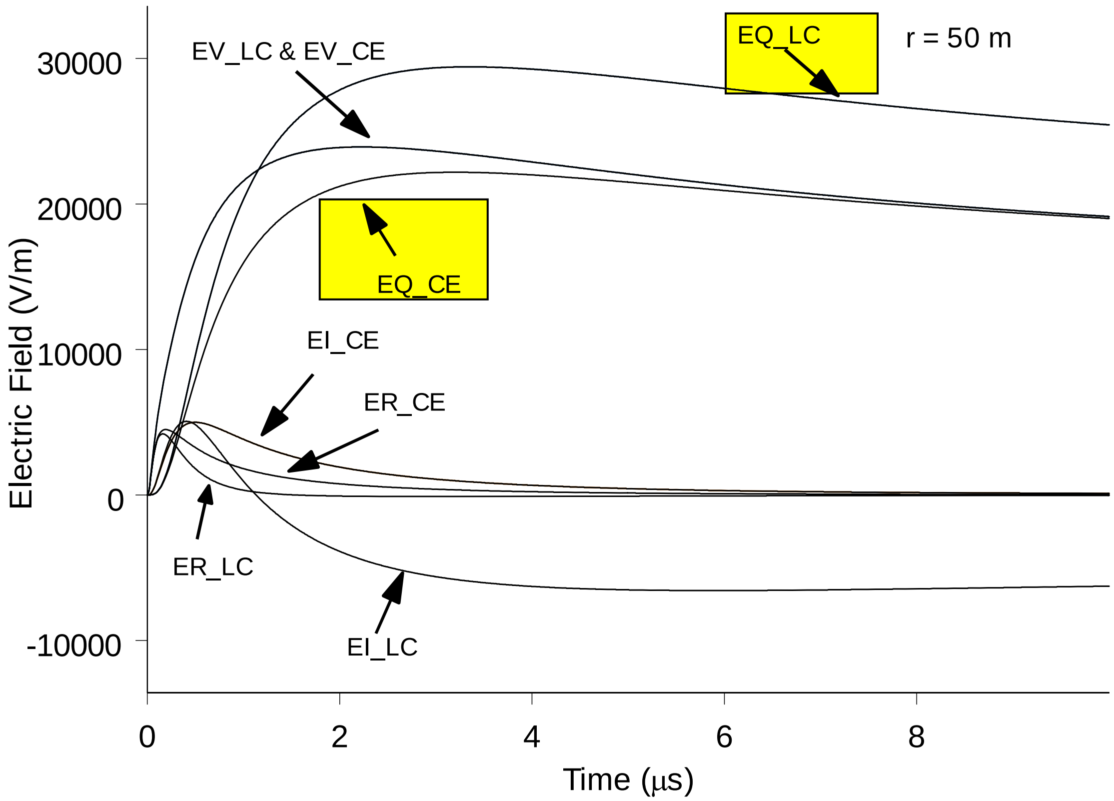
Non-uniqueness of field components (Numerical example) - 1

R = 50 m



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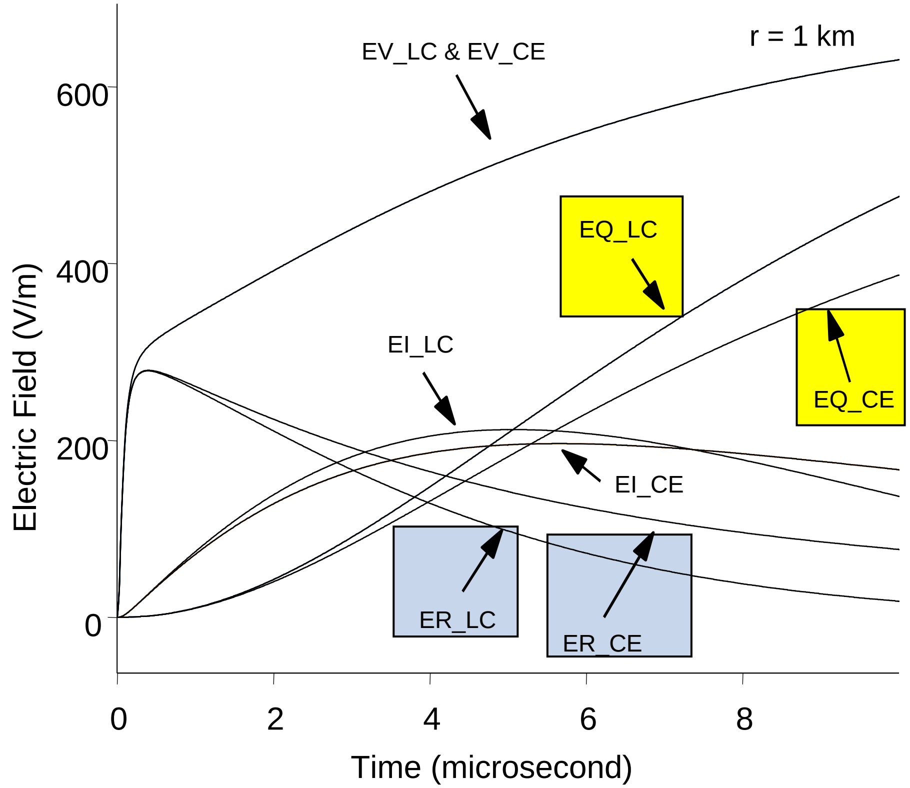
Non-uniqueness of field components (Numerical example) - 2

R = 1000 m



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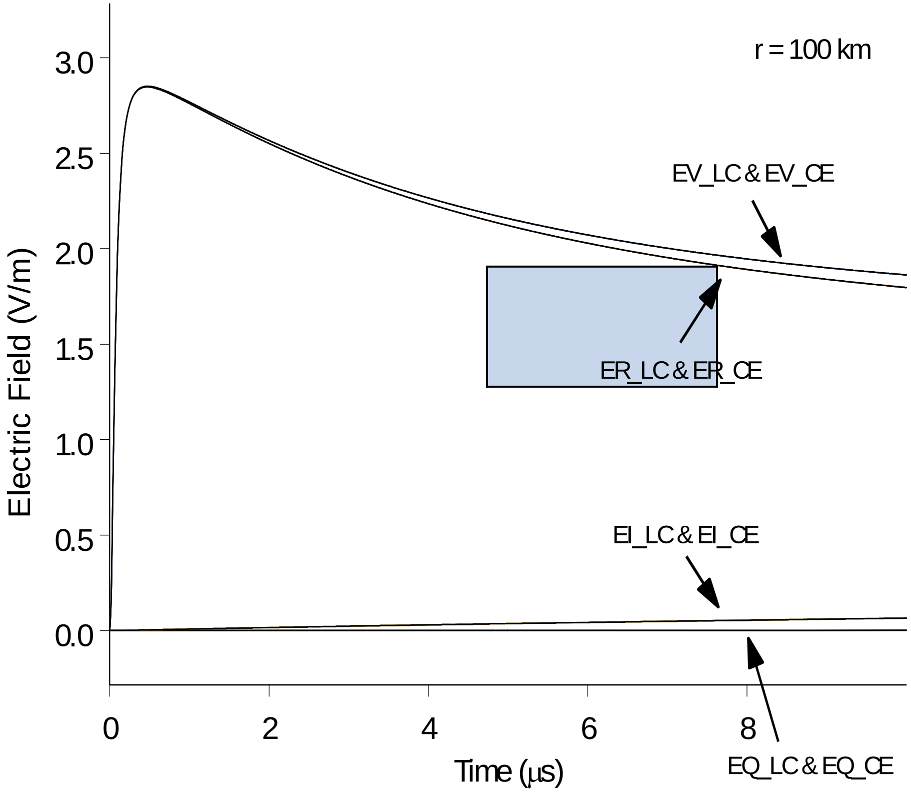
Non-uniqueness of field components (Numerical example) - 3

R = 100 000 m



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Electric field at ground plane

(Dipole method)

Both the gradient of the **scalar potential** and the time derivative of the **vector potential** contribute to the **radiation field** term.

Time derivative of the **vector potential** contribute to the induction field term.

(Monopole method)

Radiation term is completely given by the time derivative of the **vector potential**.

Electrostatic and induction terms are given completely by the gradient of the **scalar potential**

No one-to-one correspondence between field components. However, total field is the same

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Inferences

Individual field components - static, induction, and radiation - are not unique

Total electric field is unique

Differences between field components are significant at close distances and negligible at far distances

In the dipole technique the gradient of scalar potential contributes to all three electric field components

In the monopole technique technique the gradient of scalar potential contributes only to the electrostatic and induction components

Caution has to be exercised in interpreting measurement results or in making approximations in calculations

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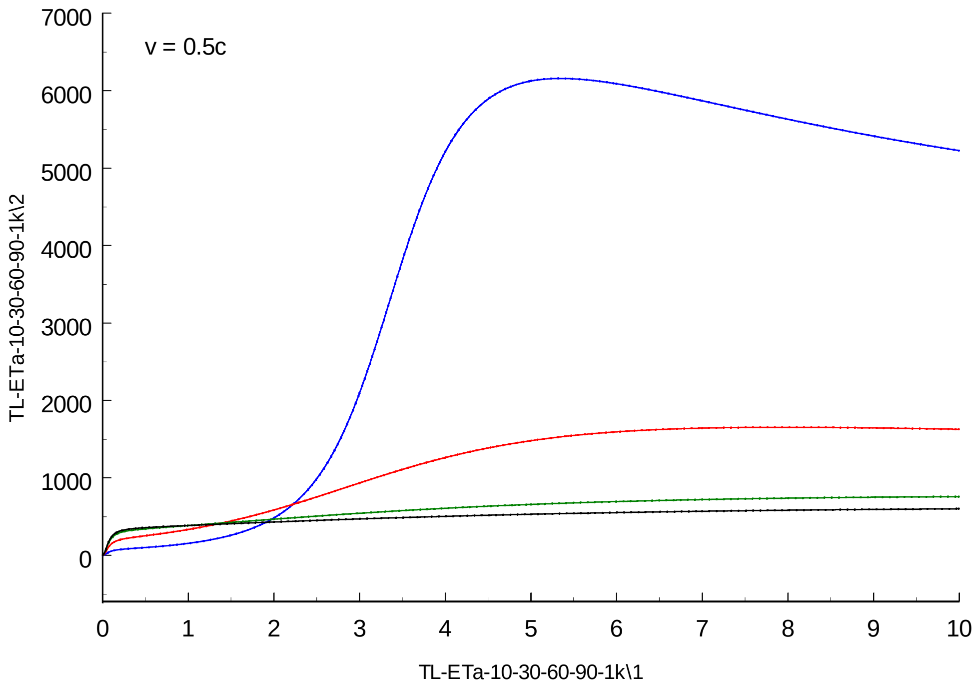
Numerical calculation – E at 1 km



The θ-component of E-field predicted by the TL model for return stroke speed v = 0.5c, above perfectly conducting ground at a distance of 1000 m from channel-base. Blue – 10º, Red – 30º, Green – 60º, Black – 90º (angle is measured with respect to vertical). Time in μs on the x-axis and E-field in V/m on the y-axis.

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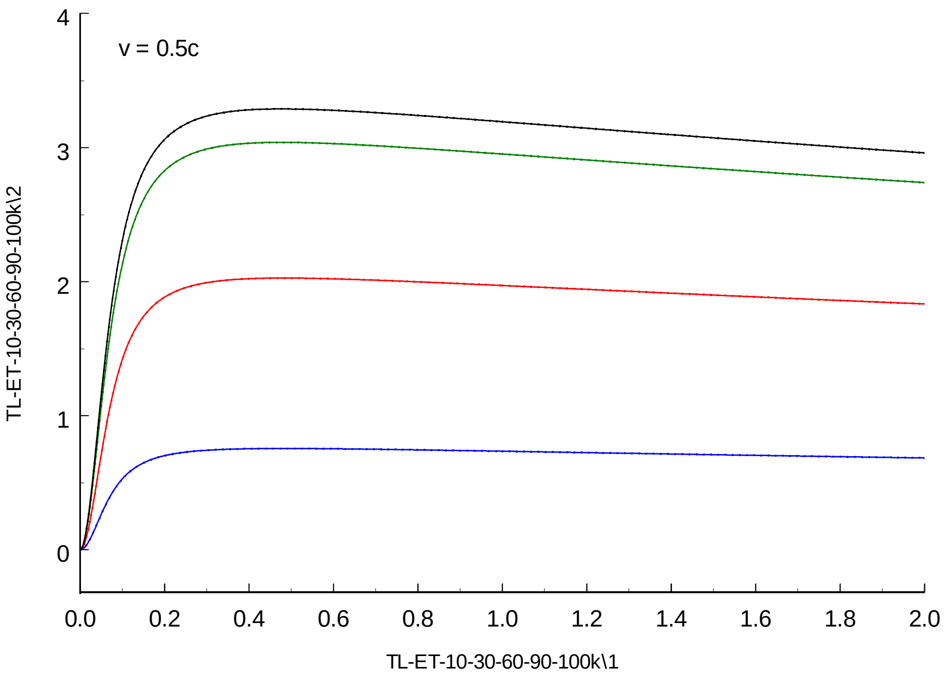
Numerical calculation – E at 100 km



The θ-component of E-field predicted by the TL model for return stroke speed v = 0.5c, above perfectly conducting ground at a distance of 100 km from channel-base. Blue – 10o, Red – 30o, Green – 60o, Black – 90o (angle is measured with respect to vertical). Time in μs on the x-axis and E-field in V/m on the y-axis.

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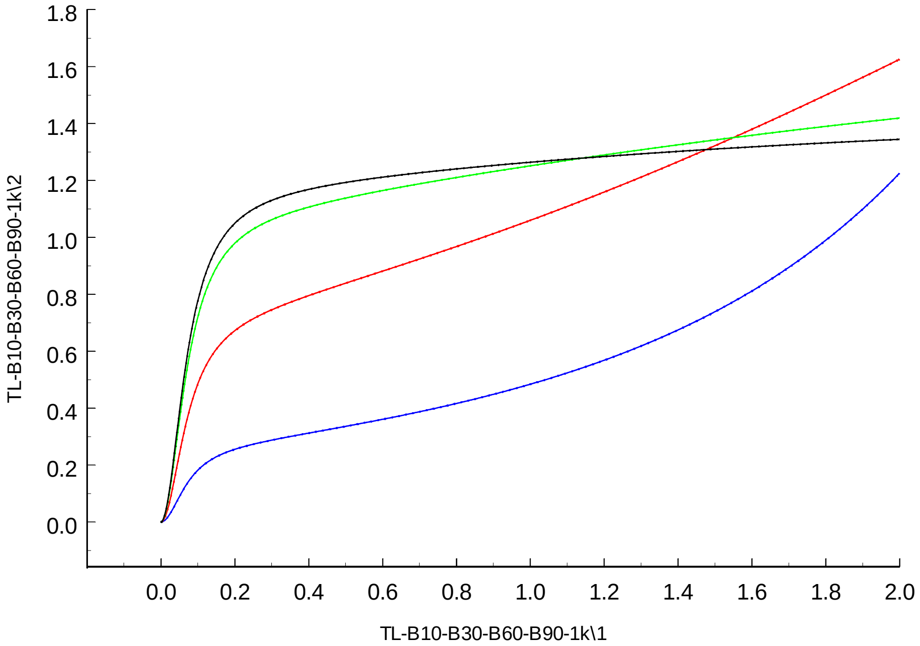
Numerical calculation – B at 1 km



The magnetic field predicted by the TL model for return stroke speed v = 0.5c, above perfectly conducting ground at a distance of 1000 m from channel-base. Blue – 10°, Red – 30°, Green – 60°, Black – 90° (angle is measured with respect to vertical). Time in μs on the x-axis and E-field in V/m on the y-axis.

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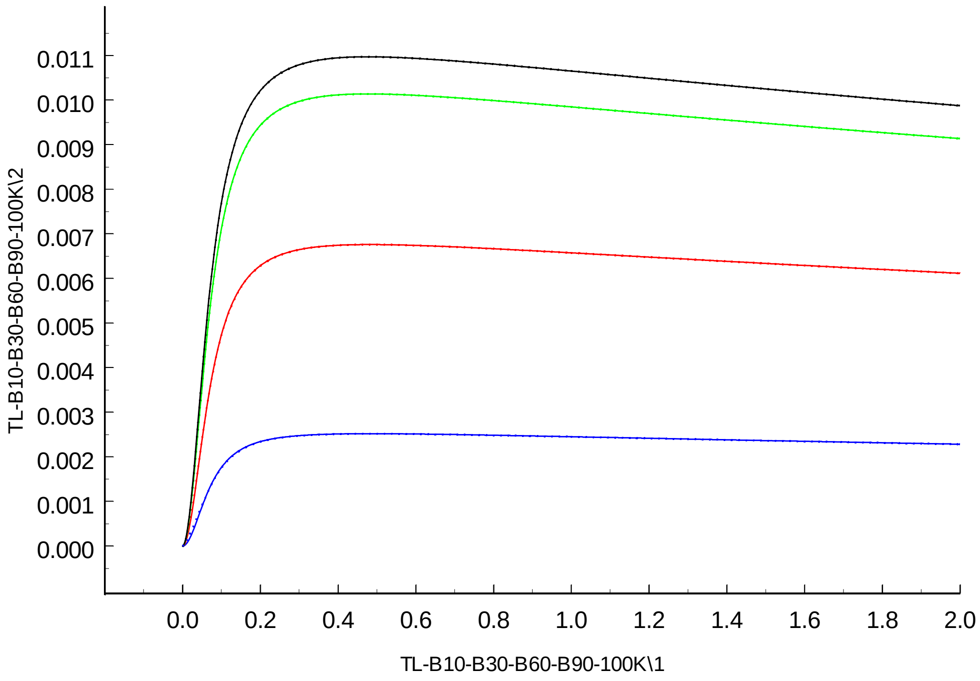
Numerical calculation – B at 100 km



The magnetic field predicted by the TL model for return stroke speed v = 0.5c, above perfectly conducting ground at a distance of 100 km from channel-base. Blue – 10o, Red – 30o, Green – 60o, Black – 90o (angle is measured with respect to vertical). Time in μs on the x-axis and E-field in V/m on the y-axis.

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Peak electric fields in V/m as a function of angle, return stroke speed and distance from channel-base.

Angle from vertical	E_θ at 100 m			E_θ at 1 km			E_θ at 100 km		
	v=0.5c	**v=0.9c**	**v=c**	**v=0.5c**	**v=0.9 c**	**v=c**	**v=0.5c**	**v=0.9c**	**v=c**
10º	73300	41800	37828	6160	3900	3783	0.76	4.9	**38**
30º	23000	14100	13137	1656	1230	1314	2.0	**7.6**	13
60º	11700	7800	7585	760	684	758	3.0	6.4	7.6
90º	9500	6700	6569	603	605	657	**3.3**	5.9	6.6

Angle from vertical	E_r at 100 m			E_r at 1 km			E_r at 100 km		
	v=0.5c	**v=0.9c**	**v=c**	**v=0.5c**	**v=0.9c**	**v=c**	**v=0.5c**	**v=0.9c**	**v=c**
10º	25000	3500	0	2690	617	0	-	-	0
30º	7280	1030	0	657	121	0	-	-	0
60º	2220	306	0	184	30	0	-	-	0
90º	0	0	0	0	0	0	0	0	0

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Electric and Magnetic Fields from a Semi-Infinite Vertical Thin-Wire Antenna Above a Conducting Plane

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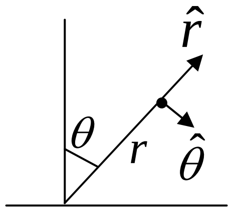
What happens if the return stroke speed is speed of light and if the current travels without any attenuation and dispersion?

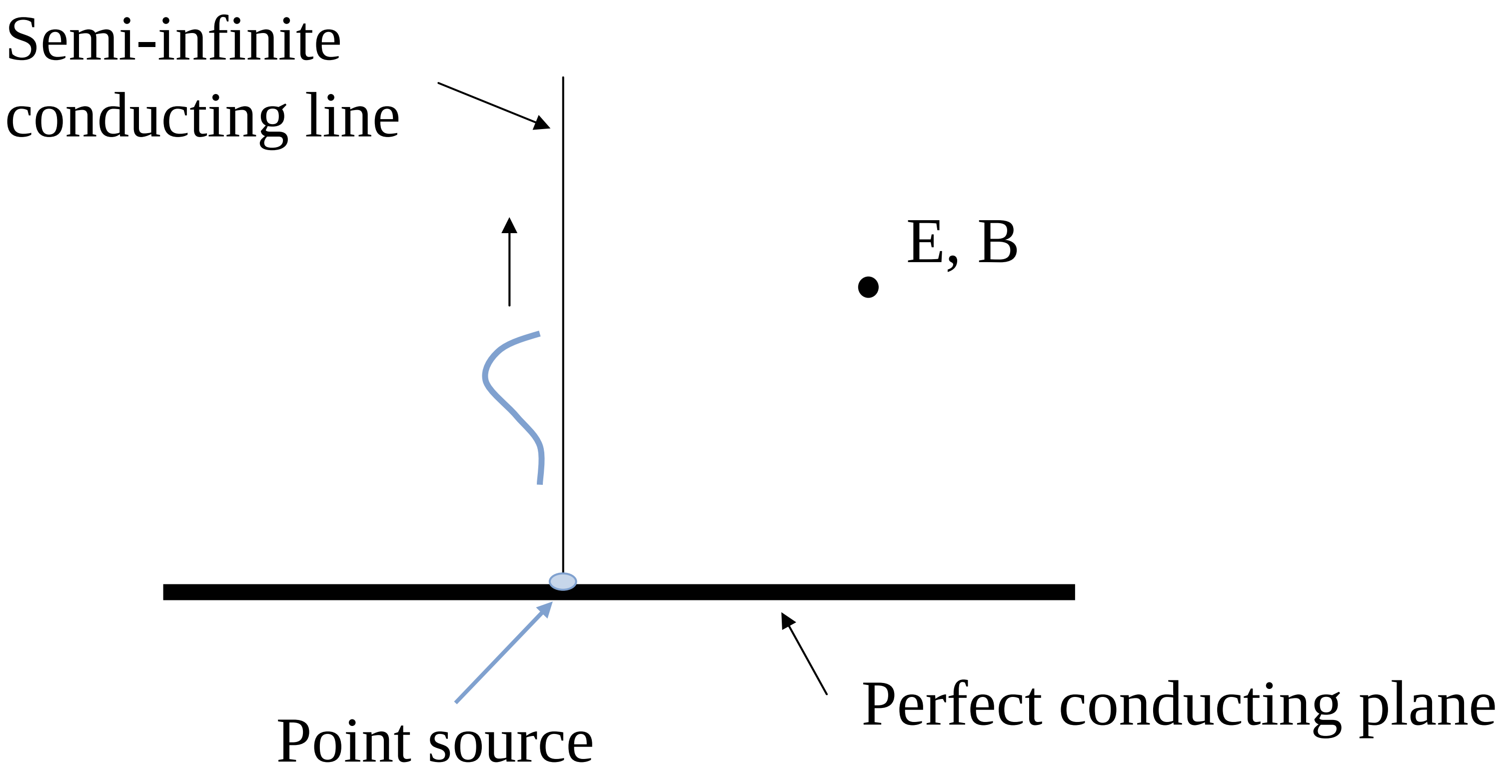
It can be proved that the exact general expression, with effect of perfect ground included, reduces to

$$\overline{E}(r,\theta,t) = \frac{1}{2\pi\varepsilon_0 c r \sin\theta} i(0, t - r/c)\hat{\theta}, \quad \theta \neq 0$$

$$\overline{B}(r,\theta,t) = \frac{1}{2\pi\varepsilon_0 c^2 r \sin\theta} i(0, t - r/c)\hat{\phi}, \quad \theta \neq 0$$



Can this structure support TEM solution?



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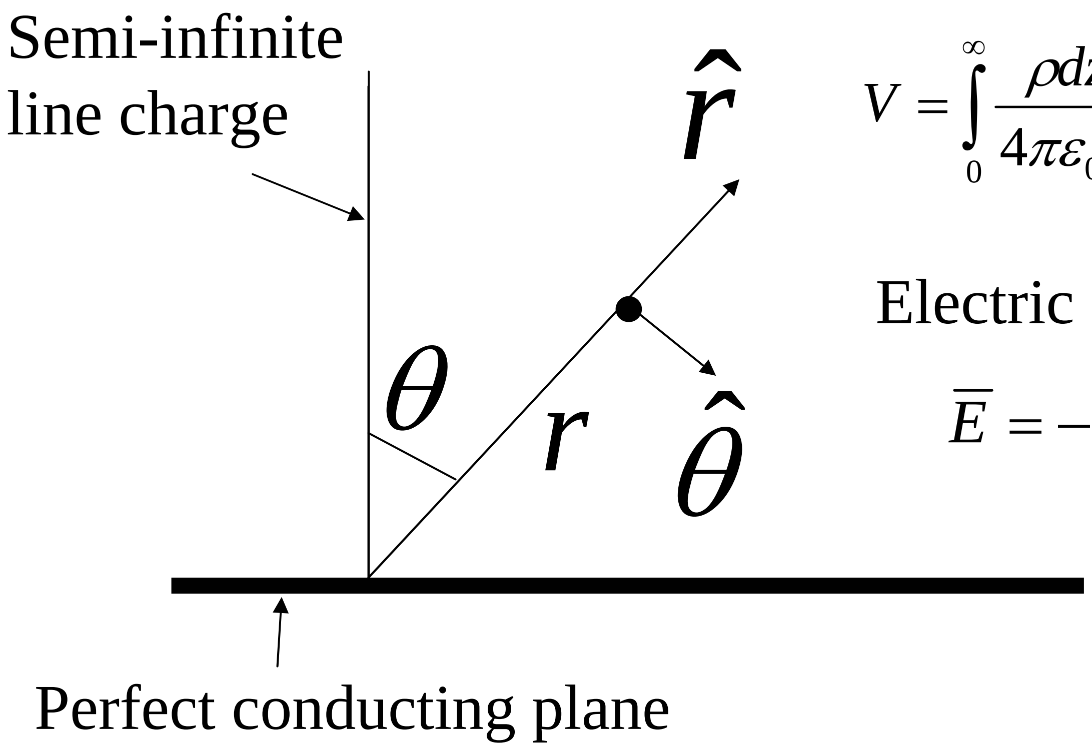
Semi-infinite uniform line charge perpendicular to a conducting plane



Scalar potential, V

$$V = \int_0^\infty \frac{\rho dz'}{4\pi\varepsilon_0 R} + \text{image}$$

Electric field (static), E

$$\overline{E} = -\nabla V$$

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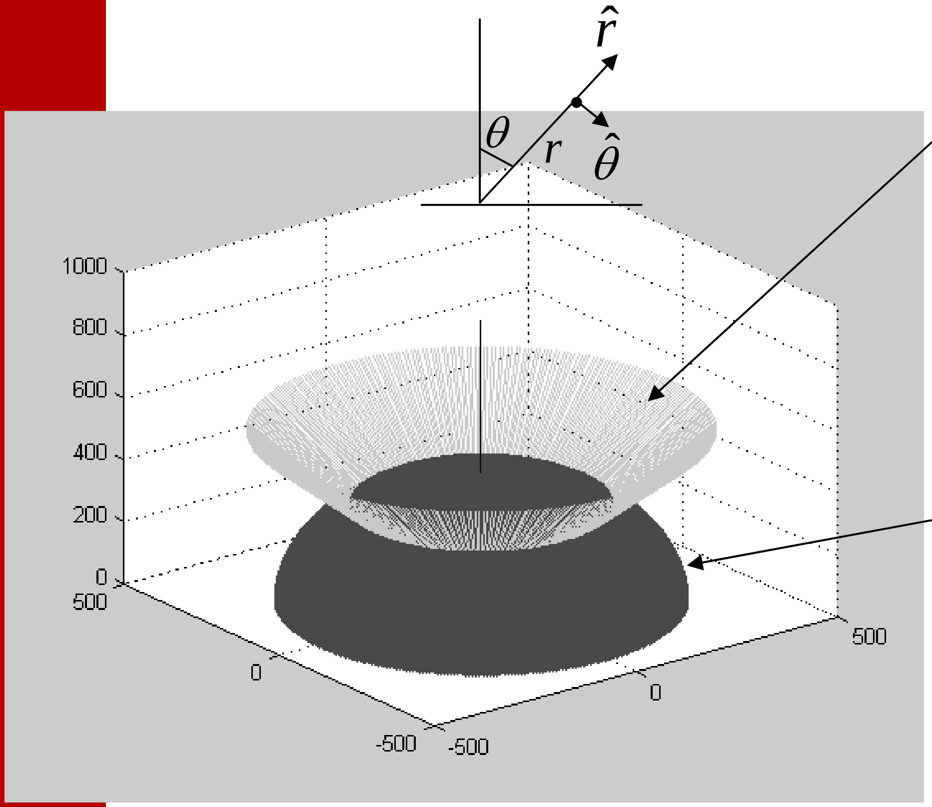
Semi-infinite uniform line charge perpendicular to a conducting plane - continued



$$V(\theta) = \frac{\rho}{4\pi\varepsilon_0}\left[\ln\frac{1+\cos\theta}{1-\cos\theta}\right]$$

(For given θ, equipotential conical surface)

$$E(r,\theta) = \hat{\theta}\,\frac{\rho}{2\pi\varepsilon_0 r\sin\theta}$$

(For given r, E-field tangential to longitudinal lines of spherical surface)

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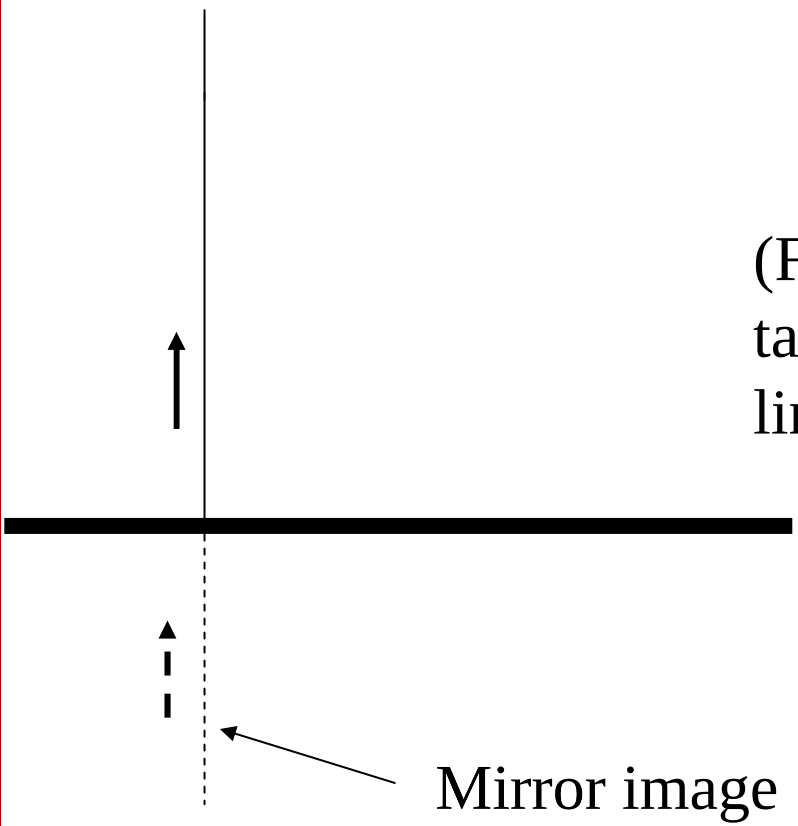
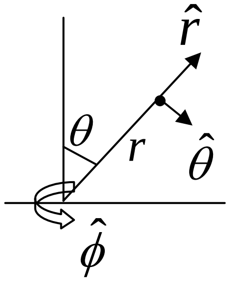
Semi-infinite uniform line current perpendicular to a conducting plane



$$B(r,\theta) = \hat{\phi}\frac{I}{2\pi\varepsilon_0 c^2 r \sin\theta}$$

(For given *r*, B-field tangential to latitudinal lines of spherical surface)



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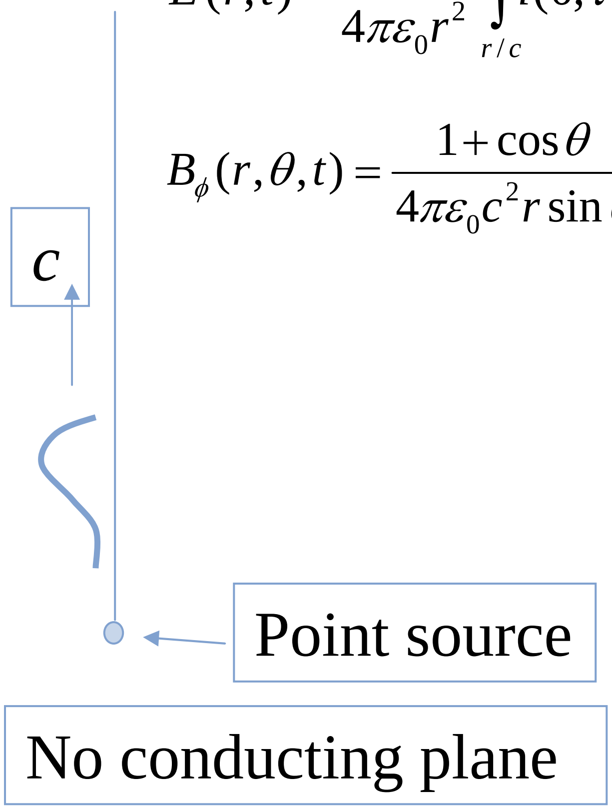
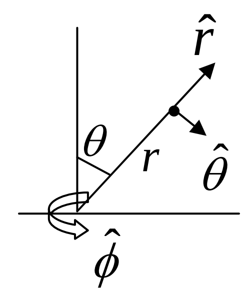
Pulse propagation on a vertical wire (exact formulation)

$$\overline{E}(r,t) = \frac{-1}{4\pi\varepsilon_0 r^2}\int_{r/c}^{t} i(0,\tau - r/c)\,d\tau\,\hat{r} + \frac{(1+\cos\theta)}{4\pi\varepsilon_0 c r \sin\theta}\, i(0,t-r/c)\,\hat{\theta}, \quad \theta \neq 0$$

$$B_\phi(r,\theta,t) = \frac{1+\cos\theta}{4\pi\varepsilon_0 c^2 r \sin\theta}\, i(0,t-r/c), \quad \theta \neq 0$$





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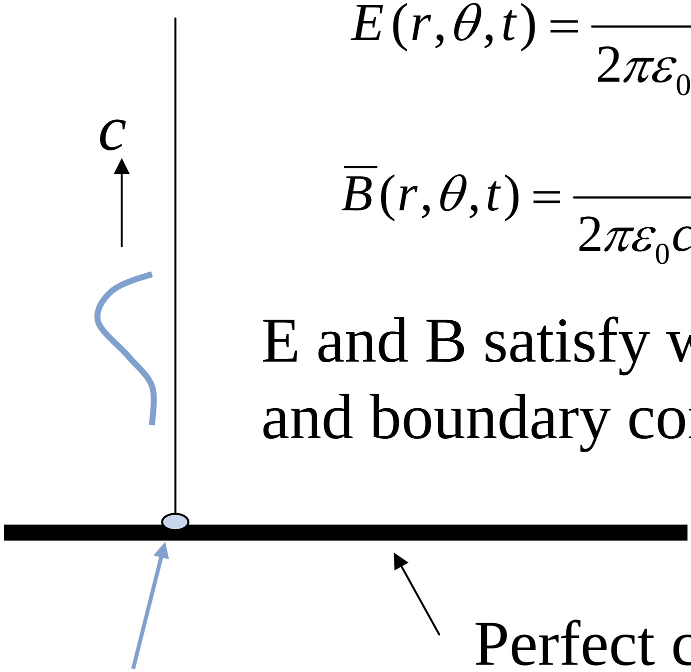
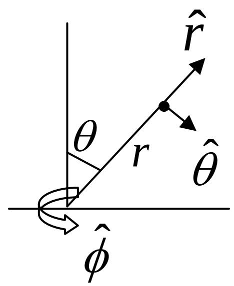
Pulse propagation on a vertical antenna (exact formulation)

$$\overline{E}(r,\theta,t)=\frac{1}{2\pi\varepsilon_0 cr\sin\theta}i(0,t-r/c)\hat{\theta},\qquad \theta\neq 0$$

$$\overline{B}(r,\theta,t)=\frac{1}{2\pi\varepsilon_0 c^2 r\sin\theta}i(0,t-r/c)\hat{\phi},\qquad \theta\neq 0$$



E and B satisfy wave equations and boundary conditions



Perfect conducting plane

Point source

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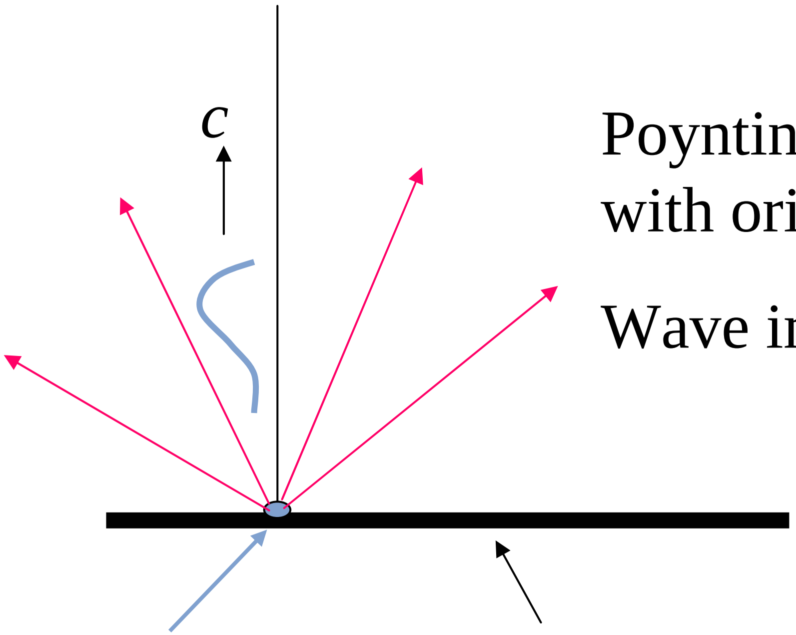
Pulse propagation on a vertical antenna (exact formulation) - continued



Poynting vectors radially-directed with origin at point charge

Wave impedance = 377 Ω

Point source   Perfect conducting plane

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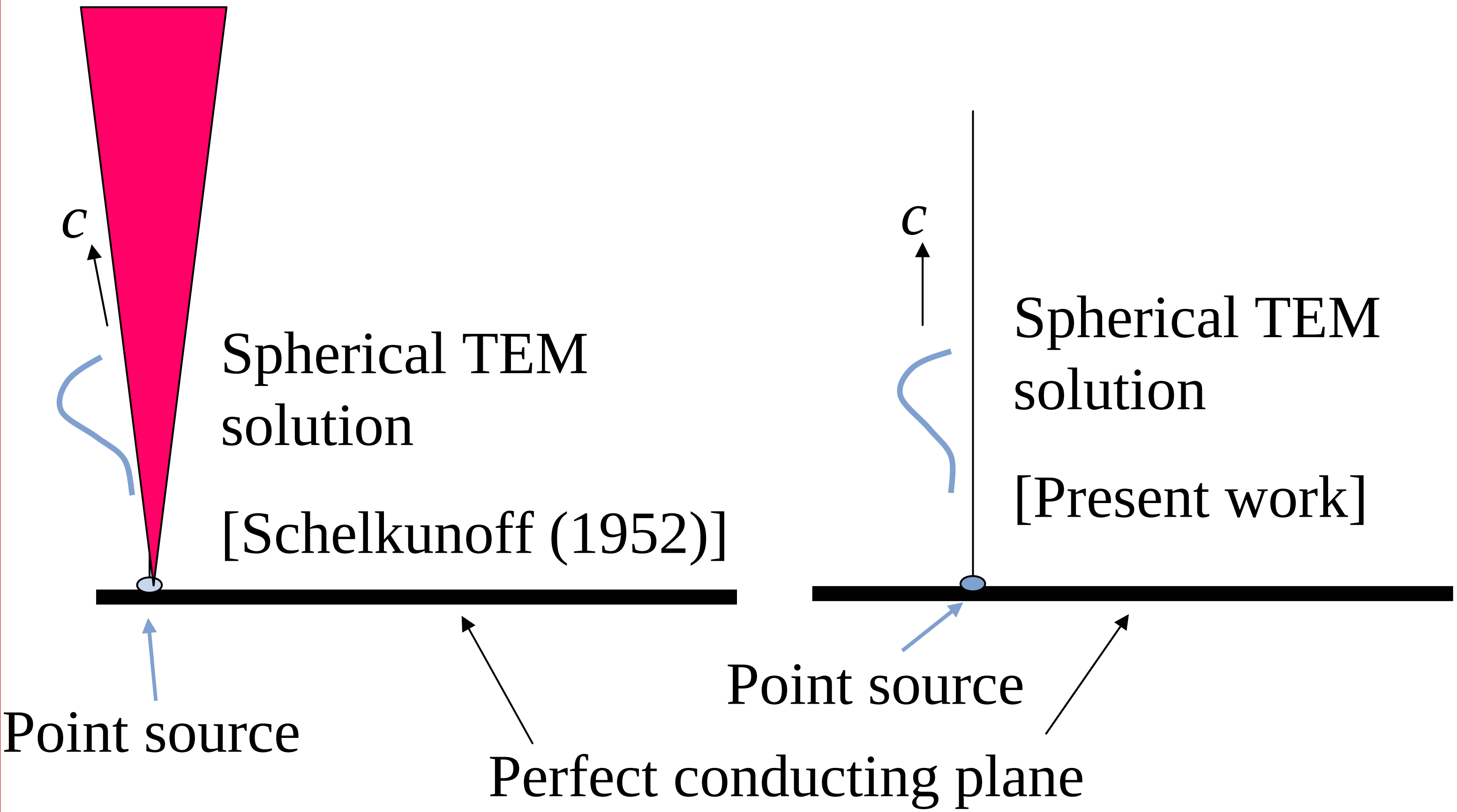
Similarity to the solution of infinite conical antenna



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Inferences

A semi-infinite conducting wire of vanishing radius perpendicular to a conducting plane, all conductors being perfect, support spherical TEM, if the only source is a point source at the bottom of the wire.

The current released from the point source travels unattenuated with the speed of light.

The Poynting vector and energy flow is in the radial direction from the source at the bottom of the antenna.

The wave impedance is the free-space impedance (377 Ω) at all distances from the antenna.

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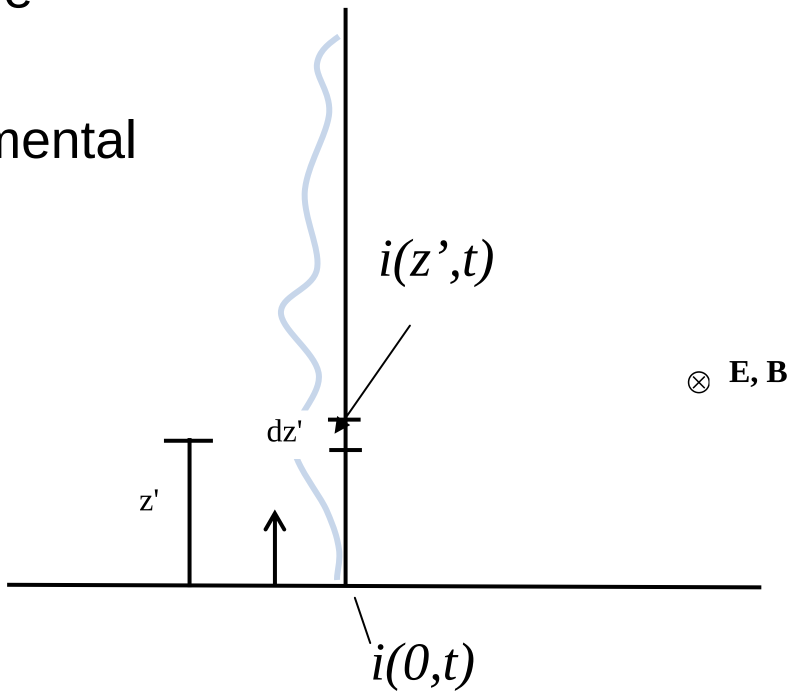
Commonly used return stroke models for computing lightning fields

Different models based on specified current distribution along the return stroke channel

Model validation using experimental data



Straight vertical channel above ground

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Common return stroke models for field computations

Bruce-Golde (Bruce and Golde, 1941)
BG

Transmission Line (Uman and McLain, 1969) TL

Travelling Current Source (Heidler, 1985)
TCS

Modified Transmission Line (Rakov and Dulzon, 1987; Nucci et al., 1988)
MTLL, MTLE

Diendorfer-Uman (Diendorfer and Uman, 1990)
DU

Modified Diendorfer-Uman (Thottappillil et al., 1991; Thottappillil and Uman, 1994)
MDU

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cost

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Inputs and outputs of the models

Inputs

- Typical current waveform representative of measured return stroke current at channel-base
- Typical optically measured return stroke speed
- Other assumptions, parameters specific to different models

Outputs

- Current distribution along the return stroke channel that varies in space and time
- Remote electric and magnetic fields calculated from the above known current distribution

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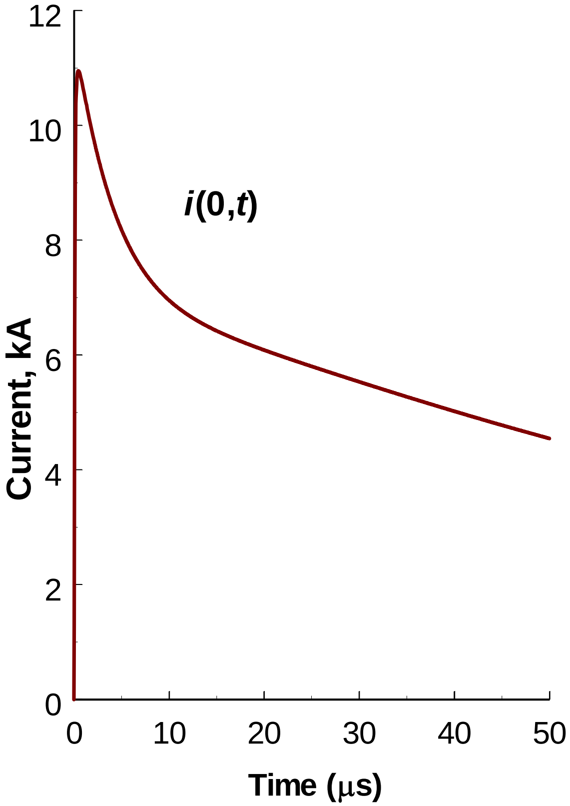
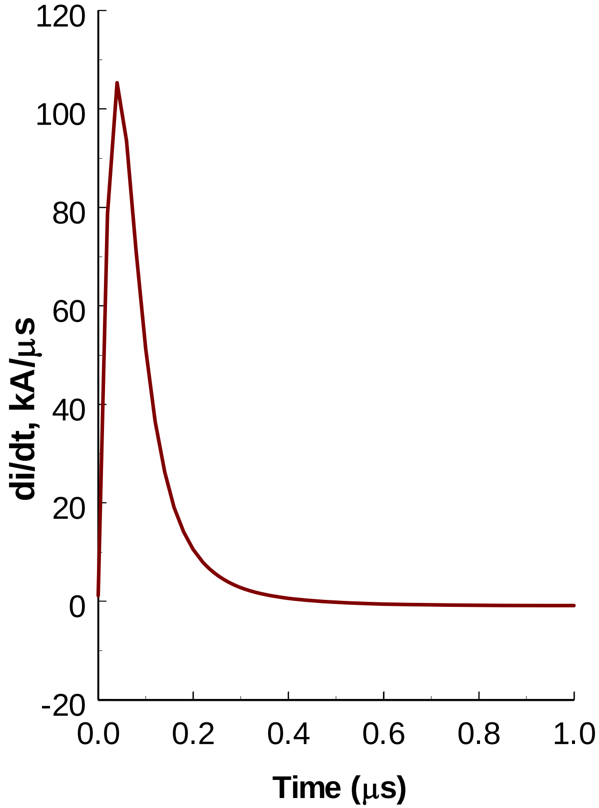
Input: Channel-base current





Adapted from Nucci et al. (1990) (Representative of typical subsequent return stroke)

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INPUT: Return stroke speed

1×10^8 m/s to 2×10^8 m/s

(one-third to two-third speed of light – the usual range of optically measured return stroke speeds)

Other parameters:

- Current decay function with height (Modified TL models)
- Discharge time constant (DU model)
- Discharge time constant and return stroke speed as functions of height (Modified DU model)

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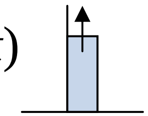
Description of models

BG: Current at ground level and along the channel behind the return stroke front the same at all instant of time

$$i(z',t) = i(0,t)$$



TL: Current at ground levels travels up the channel with the same speed as the returnstroke, without attenuation and distortion

$$i(z',t) = i(0,t-\frac{z'}{v})$$

MTLL: Same concept as TL, but the current attenuates with height as a linear function of height

$$i(z',t) = i(0,t-\frac{z'}{v})\cdot(1-\frac{z'}{H})$$

MTLE: Same concept as TL, but the current attenuates with height as an exponential function

$$i(z',t) = i(0,t-\frac{z'}{v})\cdot e^{-\frac{z'}{\lambda}}$$

(z′- height above ground, v – upward return stroke speed, H – cloud height, λ- decay constant of current with height)

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Description of models

TCS: The return stroke wavefront *instataneously* discharges the stored charge in the leader channel, and the resulting current travels down with speed of light and give rise to the current at ground ($z'=0$)

$$i(z',t) = i(0, t + \frac{z'}{c})$$

DU: Similar concept as TCS above, but instead of instantaneous discharge, an *exponential* discharge with a constant discharge time constant, τ_D

$$i(z',t) = i(0, t + \frac{z'}{c}) - i(0, \frac{z'}{v} + \frac{z'}{c}) \cdot e^{-(t - \frac{z'}{v} - \frac{z'}{c})/\tau_D}$$

MDU: Generalization of the DU model. The upward return stroke speed and the discharge time constant can be a function of height.

Other models not presented here reviewed by Rakov and Uman, 1998 and Gomes and Cooray (2000)

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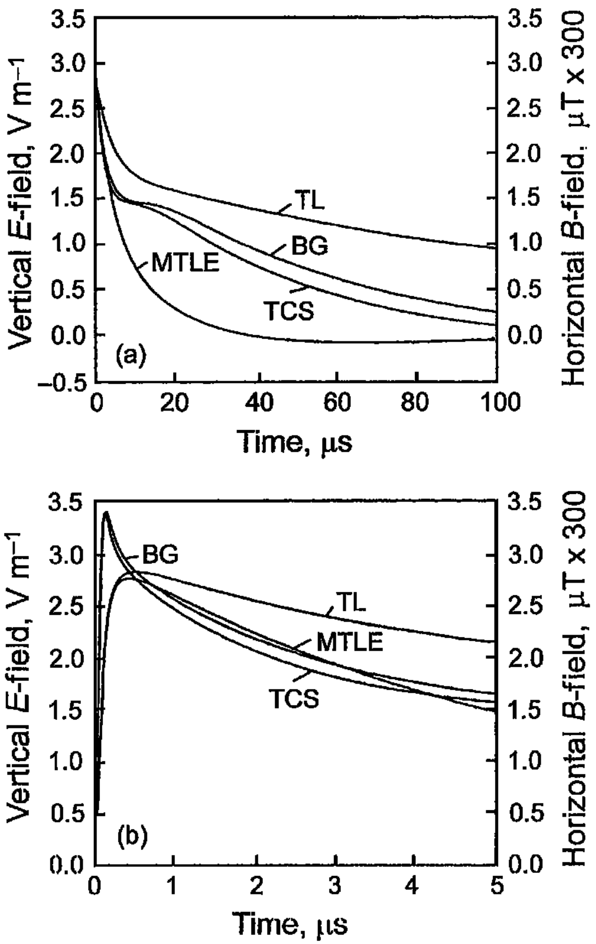
Calculated fields - 1

Calculated electric and magnetic fields from typical subsequent return stroke at a distance of 100 km, in two different time scales. [Nucci et al. (1990)]



Note the differences in the predicted fields.

The peak fields are similar to the typical measured fields at those distances.

E to B ratio equals speed of light at large distances.

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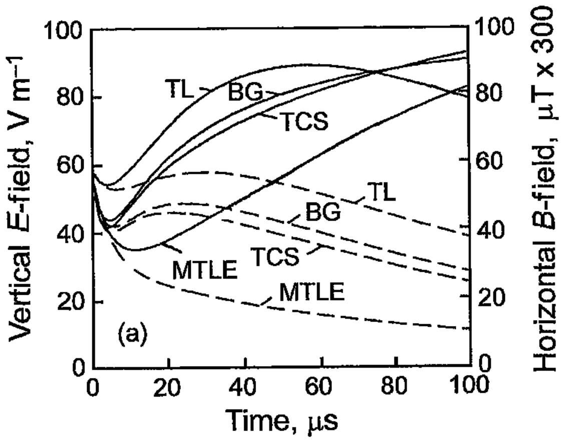
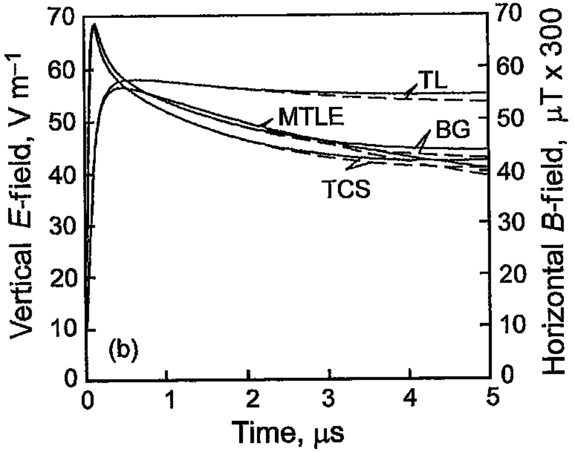
Calculated fields - 2

Calculated electric and magnetic fields from typical subsequent return stroke at a distance of 5 km, in two different time scales. Dashed lines are magnetic fields [Nucci et al. (1990)]



Note the differences in the predicted fields. The peak fields are similar to the typical measured fields at those distances. E to B ratio different from speed of light beyond the peak.



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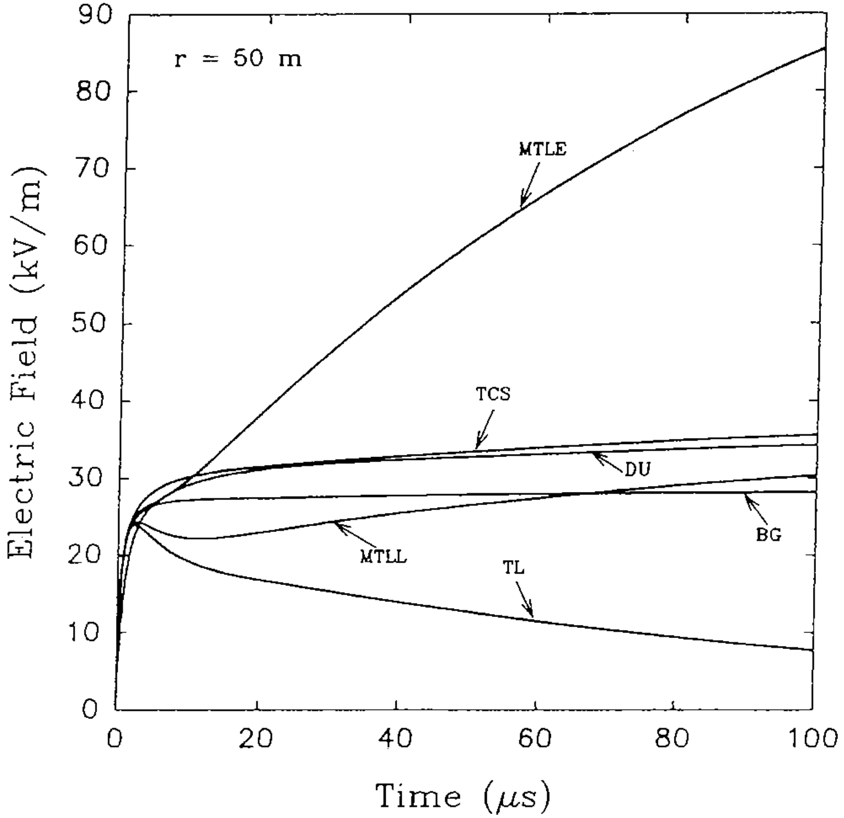
Calculated fields - 3

Calculated electric and magnetic fields from typ subsequent return strok at a distance of 50 m. [Thottappillil et al. (1997

Measured fields at similar distances have a steady value after the peak, wh is not reproduced by the TL and MTLE models



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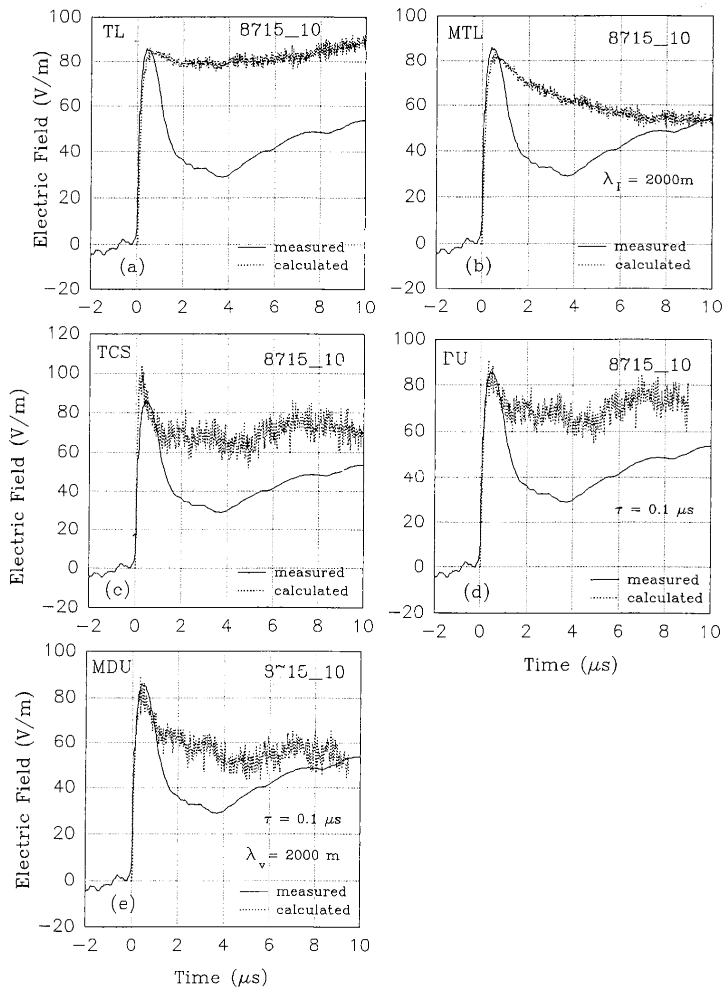
Model validation using experimental data

Model validation carried out using 18 sets of simultaneously measured channelbase current, return stroke speed and vertical electric field at about 5 km [Measurement data from Willett et al., 1989].

One example shown

Ref: Thottappillil and Uman (1993)



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Model validation using experimental data - Inferences

Absolute error in peak: $\vert E_{calc.} - E_{meas.} / E_{meas.} \vert$					
	TL	MTLE	TCS	DU	MDU
Mean error	0.17	0.16	0.43	0.23	0.21
Min. error	0.00	0.00	0.14	0.00	0.02
Max. error	0.51	0.45	0.84	0.63	0.60
Std. Deviat.	0.12	0.11	0.22	0.2	0.19

Note: MTLL model is also expected to produce a result almost the same as that of MTLE model, as far as the peak field is concerned.

All models are capable of satisfactorily predicting the peak remote electric fields, but considering its simplicity TL model preferred.

In applications where waveshapes after the peak is important, MTLL model is a good choice.

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SOME REFERENCES FOR MORE INFORMATION

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